Mathematical Aspects of Quantum Information Theory
Aspects mathématiques de la théorie de l'information quantique
(Org: **Nathaniel Johnston** (Mount Allison University) and/et **Sarah Plosker** (Brandon University))

JOSE FRANCO, University of North Florida
Matrix Means and Uncertainty Principle Functions

An *Uncertainty Principle Function* (UPF) is a function $F_\rho(A, B)$ such that

$$\mathsf{Var}_\rho(A)\mathsf{Var}_\rho(B) - |\Re(\mathsf{Cov}_\rho(A, B))|^2 \geq F_\rho(A, B).$$

In this talk, we show that when σ is a Kubo-Ando mean such that $\sigma > \sharp$, there exists a class of operators for which

$$F_\rho(A, B) := \mathsf{Var}_\rho(A)\mathsf{Var}_\rho(B) - \mathsf{Var}_\rho(A\sigma B)^2$$

is a UPF. Time permitting, we will look at several related trace inequalities.

MARK GIRARD, IQC
Convex analysis for quantum resource theories

The study of quantum resource theories encompasses many physical properties of quantum systems that can be viewed as resources and that can be used and manipulated within certain restrictions. In studying any resource theory, one aims to find useful resource measures and other conditions that can be used to express when conversions between resources is possible within the framework. Since many resource theories arising in quantum information (such as those of quantum entanglement and coherence) exhibit convex structure that can be exploited in developing a mathematical framework for understanding, tools from convex analysis can be applied to aid in our understanding. Here we investigate some techniques using convex optimization theory that can be applied to convex resource theories, and provide explicit examples to certain resource theories such as that of entanglement.

NATHANIEL JOHNSTON, Mount Allison University
The Spectra of Entanglement Witnesses

We consider the problem of characterizing the possible spectra (i.e., multisets of eigenvalues) of entanglement witnesses. We completely solve this problem in the two-qubit case and we derive a large family of new necessary conditions on the spectra in arbitrary dimensions. We also establish a natural duality relationship with the set of absolutely separable states, and we completely characterize witnesses (i.e., separating hyperplanes) of that set when one of the local dimensions is 2.

KARL-PETER MARZLIN, St Francis Xavier University
Causal perturbation theory in quantum optics

Like other quantum field theories, the theory of atoms interacting with light requires renormalization to handle divergent terms. For relativistic field theories, Epstein and Glaser have proposed causal perturbation theory (CPT) as a method to avoid divergences before they occur. The key part of this method is to employ proper distribution splitting to define retarded and advanced propagators. In this talk, I will describe how CPT can be used to describe spontaneous emission of light by two-level atoms. [J. Math. Phys. 59, 042103 (2018)]

ABEL MOLINA, IQC, UW
The optimality of projections for quantum state exclusion

We will first motivate the problem of quantum state exclusion of pure states, through its connections with the PBR game and with compatibility conditions for quantum state assignments. Then, we will discuss our result regarding the optimality of projections for perfect state exclusion of 3 pure states in 3 dimensions (arXiv:1702.06449). We will describe our techniques to prove this result, which are based on arguments involving convexity, rank and symmetry properties. Finally, we will discuss possible avenues for extending our results.

SARAH PLOSKER, Brandon University
Switching the hypercube while maintaining perfect state transfer

In our quest to find new graphs with perfect state transfer (a desirable property in quantum information theory), we perform Godsil-McKay switching on the hypercube to create graphs that maintain many of the same properties of the hypercube. Further perturbations are also studied.

JITENDRA PRAKASH, University of Waterloo
Entanglement breaking rank

A quantum channel is entanglement breaking if and only if it admits a Choi-Kraus representation consisting of rank-one Choi-Kraus operators. We define the entanglement breaking rank of an entanglement breaking channel to be the least number of such rank-one operators required in its Choi-Kraus representation. We show that the problem of computing the entanglement breaking of the channel:

$$X \mapsto \frac{1}{d+1}(X + \mathsf{Tr}(X)I_d),$$

is equivalent to the existence problem of SIC POVM in dimension d.

MIZANUR RAHAMAN, University of Waterloo
Eventually Entanglement Breaking Maps

The set of Entanglement breaking maps is one of the central aspects in the study of quantum information science and also in the theory of completely positive maps. In this talk, I will present a certain class of linear maps on matrix algebras that have the property that they become entanglement breaking after composing finite or infinite number of times with themselves. These maps are called eventually entanglement breaking. This means that the Choi matrix of the iterated linear map becomes separable in the tensor product space. It turns out that the set of eventually entanglement breaking maps forms a rich class within the set of all completely positive maps. The motivation of this work is the "PPT-squared conjecture" made by M. Christandl that says every PPT channel, when composed with itself, becomes entanglement breaking. In this work, it is proved that every unital PPT-channel becomes entanglement breaking after finite number of iterations. This is a joint work with Sam Jaques and Vern Paulsen

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JAMIE SIKORA, Perimeter Institute for Theoretical Physics
A fidelity measure for quantum strategies

We introduce a definition of the fidelity function for multi-round quantum strategies, called the strategy fidelity, which is a generalization of the fidelity function for quantum states. We provide many interesting properties of the strategy fidelity including a Fuchs-van de Graaf relationship with the strategy norm. We also provide a very general monotonicity result for both the strategy fidelity and strategy norm under the actions of strategy-to-strategy linear maps. We illustrate an operational interpretation of the strategy fidelity in the spirit of Uhlmann's Theorem and discuss its application to the security analysis of two different quantum cryptographic tasks.

This is joint work with Gus Gutoski and Ansis Rosmanis.

DEPING YE, Memorial University of Newfoundland
The ubiquity of Einstein's "spooky action at a distance"

Quantum Entanglement, also called "spooky action at a distance" by Einstein, plays fundamental roles in the development of quantum information theory. The ubiquity of quantum entanglement has been a long-standing question in quantum information theory related areas. In this talk, I will present the phase transition phenomenon of quantum entanglement, in particular the threshold where the phenomenon occurs, and hence provided a solution to the long-standing question regarding the ubiquity of quantum entanglement. This talk is based on a joint work with Aubrun and Szarek (Comm. Pure Appl. Math., 2014)