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A dynamical approach to semilinear elliptic equations

In this talk I will describe a new procedure for reducing a semilinear elliptic PDE on a bounded domain $\Omega \subset \mathbb{R}^n$ to an infinite-dimensional dynamical system on the boundary $\partial\Omega$.

Suppose u satisfies the equation $\Delta u + F(x,u)$ on Ω. When the domain is deformed through a one-parameter family $\{\Omega_t\}$, the Cauchy data of u on $\partial\Omega_t$ will satisfy a first-order evolution equation. This equation is ill-posed, in the sense that it does not admit solutions forwards or backward in time for generic initial data. However, if the domain is deformed smoothly to a point, this equation admits an exponential dichotomy, so there exist two distinguished subspaces of boundary data (at each time t) for which solutions exists forward and backward in time, respectively. When the PDE is selfadjoint, the evolution equation will be Hamiltonian, so the unstable subspace is Lagrangian and hence has a well defined Maslov index. These constructions generalize previous work in spatial dynamics, which considered elliptic equations on cylindrical domains.

This is joint with with Margaret Beck, Christopher Jones, Yuri Latushkin and Alim Sukhtayev.