
Ergodic Theory, Dynamical systems, Fractals and Applications
Théorie ergodique, systèmes dynamiques, fractals et applications
(Org: **Shafiqul Islam** (University of Prince Edward island) and/et **Franklin Mendivil** (Acadia University))

ARNO BERGER, University of Alberta
Classifying linear flows (again)

Finite-dimensional linear flows are among the simplest dynamical systems imaginable. When exactly do two such flows have the same phase portrait, up to a change of coordinates? If the change of coordinates and rescaling of time are differentiable, this question is easy to answer; otherwise, it is much more delicate. Nevertheless, complete answers have been given, independently, by N. Kuiper and N. Ladis in the 1970s. Although the ensuing topological classification of linear flows is easy to state, the basic ideas behind it have not found their way into textbooks. This talk attempts to rectify this. A new, completely elementary proof of the classification theorem is presented which only requires basic linear algebra. As an aside, several inaccuracies in the classical arguments will be corrected. (Joint work with A. Wynne.)

MATT BETTI, York University
A Dynamical Systems Approach to Mitigating Honey Bee Disease

Honey bee colonies have been in steady decline over the past few decades. This decline is thought to be caused by a number of factors which act together to create hostile conditions for honey bees. Here, we use dynamical systems to study the influence of disease and seasonality on honey bee colonies and use the predictions from the models to inform prevention strategies. We analyze stability of healthy colonies, conditions for invasion, and the stabilizing effects of feedback loops in these models.

ILLIA BINDER, University of Toronto
KdV equation: Deift conjecture and Dubrovin Flow

In 2008, P. Deift conjectured that the solution of KdV equation with almost periodic initial data is almost periodic in time. I will discuss the proof of this conjecture in the case of the so-called Sodin-Yuditskii type initial data, i.e. the initial data for which the associated Schroedinger operator has purely absolutely continuous spectrum which satisfies certain thickness conditions. In particular, we establish the existence, uniqueness, and almost periodicity of the solutions with small analytic quasiperiodic initial data with Diophantine frequency vector. A careful analysis of the Dubrovin flow for the initial data plays a crucial role in the proof. This is a joint work with D. Damanik (Rice), M. Goldstein (Toronto) and M. Lukic (Rice).

ERIK BOLLT, Clarkson University
On Matching, and Even Rectifying, Dynamical Systems through Koopman Operator Eigenfunctions

Matching dynamical systems, through different forms of conjugacies and equivalences, has long been a fundamental concept, and a powerful tool, in the study and classification of non-linear dynamic behavior (e.g. through normal forms). In this presentation we will argue that the use of the Koopman operator and its spectrum is particularly well suited for this endeavor, both in theory, but also especially in view of recent data-driven algorithm developments. We believe, and document through illustrative examples, that this can nontrivially extend the use and applicability of the Koopman spectral theoretical and computational machinery beyond modeling and prediction, towards what can be considered as a systematic discovery of “Cole-Hopf-type” transformations for dynamics.

CHRISTOPHER BOSE, University of Victoria
Mixing for random maps with a common indifferent fixed point

This talk will describe ongoing work about mixing rates for random intermittent maps. New results in both the annealed mixing (average over the randomizing space) and quenched mixing (for a.e. fixed randomization) cases will be described. Although this is a large project involving many hands, I will focus mainly on recent joint work with W. Bahsoun and M. Ruziboev (Loughborough) and A. Quas and M. Tanzi (Victoria).

TRUBEE DAVISON, University of Colorado

Unitary representations of the Baumslag-Solitar group associated to the Cantor Set

The Cantor Set supports a Borel probability measure known as the Hutchinson measure which satisfies a well known fixed point relationship. Previously it has been shown by P. Jorgensen and D. Dutkay that the Cantor set can be extended to an inflated Cantor set, \mathcal{R} , on a subset of the real line, which supports an extended Hutchinson measure μ . Unitary dilation and translation operators can be defined on $L^2(\mathcal{R}, \mu)$ which satisfy the Baumslag-Solitar group relation, and give rise to a multi-resolution analysis. The filter function associated to this construction can be used to produce a measure, m , on the solenoid, \mathcal{S} , a compact topological group. The Hilbert space $L^2(\mathcal{S}, m)$ also has a unitary representation of the Baumslag-Solitar group, and there exists a generalized Fourier transform between $L^2(\mathcal{R}, \mu)$ and $L^2(\mathcal{S}, m)$. In this talk, we will build off of Jorgensen and Dutkay's work and show that the unitary operators on $L^2(\mathcal{S}, m)$ mentioned above are related to each other via a family of partial isometries, which satisfy some interesting properties.

CHRISTOPHER ESSEX, Applied Mathematics, University of Western Ontario

The Entropy Production Paradox: The Mysterious World Between Diffusion and Waves.

The diffusion and wave equations are iconic symbols of irreversible and reversible processes (i.e. "not undoable" and "undoable" respectively). These physical properties are reflected in their intrinsic mathematical structures. But what happens if one finds a way to smoothly bridge the gap between these dichotomous extremes? With fractional calculus one can! Many strange things emerge, not the least of which is "the entropy production paradox," which works contrary to fundamental intuitions.

MARLENE FRIGON, Universite de Montreal

Existence and multiplicity results for systems of differential equations

In this talk, we will present existence and multiplicity results for systems of first order differential equations. One should point out that one can find very few multiplicity results for systems of differential equations in the literature. Our results will rely on a new notion called the method of solution-region. This notion generalizes the method of upper and lower solutions in the scalar case.

IGNACIO GARCIA, Universidad Nacional de Mar del Plata and CONICET

Assouad dimension of self-similar sets with overlaps in \mathbb{R}^d

It was shown by Fraser, Henderson, Olson and Robinson that, in \mathbb{R} , the Assouad dimension of non trivial self-similar sets satisfies the dichotomy: if the weak separation property (WSP) holds then the Hausdorff and Assouad dimensions of the set coincide, while if WSP is not satisfied, then the Assouad dimension is 1. In \mathbb{R}^d , they show that if the self-similar set is in general position and the WSP is satisfied, then Hausdorff and Assouad dimensions still coincide, while if WSP does not hold then the Assouad dimension is at least 1. For this class of sets, we present a formula for the Assouad dimension that considers the overlapping directions. A lower bound for the Assouad dimension of the set is given by the dimension of the vector space spanned by these directions, with strict inequality if the dimension is smaller than d .

SHAFIQL ISLAM, University of Prince Edward Island (UPEI)

A General Piecewise Spline Maximum Entropy Method for Position Dependent Random Maps

Let $\{\tau_1, \tau_2, \dots, \tau_K\}$ be a collection of nonsingular maps on $[0, 1]$ into $[0, 1]$ and $\{p_1(x), p_2(x), \dots, p_K(x)\}$ be a collection of position dependent probabilities on $[0, 1]$ into itself. We consider position dependent random maps $T = \{\tau_1, \tau_2, \dots, \tau_K; p_1(x), p_2(x), \dots, p_K(x)\}$

such that T preserves a unique absolutely continuous invariant measure μ^* with density f^* . In this talk, we describe a general piecewise spline maximum entropy method for the approximation of f^* . We present a proof of convergence of the general piecewise spline maximum entropy method for position dependent random maps. We also present numerical examples.

KAMRAN KAVEH, Harvard University
Games of multicellularity

Evolutionary game dynamics are often studied in the context of different population structures. Here we propose a new population structure that is inspired by simple multicellular life forms. We study deterministic evolutionary dynamics with mutations, and derive exact conditions for selection to favor one strategy over another. Our main result has the same symmetry as the well-known sigma condition, which has been proven for stochastic game dynamics and weak selection. We discuss evolutionary stability condition (ESS) in Prisoner's Dilemma and Hawk-Dove games as examples of this model.

FRANKLIN MENDIVIL, Acadia University
A V -variable approach to fractal image compression

Most fractal image compression methods rely on variations of Jacquin's fractal "block coding" algorithm. In his approach, the image is first partitioned twice, once into "large" blocks and again into "small" blocks; the small blocks are typically one-half the size of the large blocks. Given these two block partitions, the algorithm works by scanning through all the small blocks and, for each one, searching the large blocks to find the "best" match when suitably transformed. The theory underlying fractal block coding is that of "local" IFS fractals.

In this talk, I will discuss a completely new fractal-based algorithm for image compression, one which is inspired by the theory of V -variable fractals. V -variable fractals are intuitively fractals with at most V different "forms" at each level of magnification. In the context of a block partition, this means that for a given size of block, there are at most V different blocks of that size.

Our algorithm also works with a block decomposition of the image and progresses down the size scale. However, we define no contractive mappings from larger scale to smaller scale, but instead we sort and cluster blocks of a given size to find V "representatives." The inspiration from V -variable fractals is in the way the structure of the image is encoded into the V -variable tree representation. At each size "level" there are only V types and so we need only store how each type at level n is composed of the types at level $n + 1$.

DORETTE PRONK, Dalhousie University
Metrics on Fractals Defined Using Equational Systems

In his 2011-paper in *Advances of Mathematics*, Leinster introduced equational systems as an algebraic way to describe the topology of a self-similar fractal. This approach applies to IFS-fractals satisfying the open set condition, but also fractafolds and some Julia sets are included. However, the fractals are considered as objects on their own, without the embedding into Euclidean space. Also, the metric is not part of the data.

In this talk I will give an overview of Leinster's approach and introduce some ways of adding a metric to fractals defined in this way. This is joint work with James Eckstein and Robert Dawson.

S M ASHRAFUR RAHMAN, York University
*A mechanistic dose-response model of *Listeria monocytogenes* infection in human populations*

The utility of characterizing the effects of strain variation and individual/subgroup susceptibility on dose-response outcomes has motivated the search for new approaches beyond the popular use of the exponential dose-response model for listeriosis. While descriptive models can account for such variation, they have limited power to extrapolate beyond the details of particular outbreaks. In contrast, this study exhibits dose-response relationships from a mechanistic basis, quantifying key biological factors involved in pathogen-host dynamics. An efficient computational algorithm and geometric interpretation of the infection pathway are developed to connect dose-response relationships with the underlying bistable dynamics of the model. Relying on

in-vitro experiments as well as outbreak data, we estimate plausible parameters for the human context. Despite the presence of uncertainty in such parameters, sensitivity analysis reveals that the host response is most influenced by the pathogen-immune system interaction. In particular, we show how variation in this interaction across a subgroup of the population dictates the shape of dose–response curves.

TORU SERA, Kyoto University

Generalized arcsine laws for interval maps with indifferent fixed points

We present a distributional limit theorem for the occupation ratio measures of interval maps with a finite number of indifferent fixed points. This limit theorem is a multiray extension of Thaler’s generalized arcsine laws [3] for interval maps with two indifferent fixed points, and is inspired by studies (e.g., [1]) of occupation times of diffusion processes on multiray. This talk is based on a joint work [2] with Kouji Yano (Kyoto University).

References

[1] M. Barlow, J. Pitman and M. Yor, Une extension multidimensionnelle de la loi de l’arc sinus, in *Séminaire de Probabilités, XXIII*, 294–314, Lecture Notes in Math., vol. 1372, Springer, Berlin, 1989.

[2] T. Sera and K. Yano, Multiray generalization of the arcsine laws for occupation times of infinite ergodic transformations, Preprint available at arXiv:1711.03260.

[3] M. Thaler, A limit theorem for sojourns near indifferent fixed points of one-dimensional maps, *Ergodic Theory Dynam. Systems* **22**, no. 4, 1289–1312, 2002.

SASCHA TROSCHEIT, University of Waterloo

Self-conformal sets with positive Hausdorff measure

We investigate the Hausdorff measure and content on a class of quasi self-similar sets that include self-conformal sets. We show that any Hausdorff measurable subset of such sets has comparable Hausdorff measure and Hausdorff content. In particular, we show that graph-directed and sub self-conformal sets with positive Hausdorff measure are Ahlfors regular, irrespective of separation conditions. We use this to resolve a self-conformal extension of the dimension drop conjecture for self-conformal subsets of the line with positive Hausdorff measure by showing that its Hausdorff dimension falls below the expected value if and only if there are exact overlaps.

MICHAEL YAMPOLSKY, University of Toronto

Rotation domains of analytic maps

The study of Siegel disks has emerged as one of the key topics in modern low-dimensional dynamics. I will review some recent results on the geometry of Siegel disks of one- and several-dimensional polynomial maps.