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Existence and Regularity of Solutions to Some Singular Parabolic Systems

Recently, authors have studied the existence and regularity of solutions to systems of partial differential equations featuring singular nonlinearities. Under homogeneous Dirichlet boundary conditions, these reaction terms may grow singular as one approaches the boundary. Equations of this form have applications in biology and physics, specifically in the study of enzyme kinetics and the study of the gravitational potential of self gravitating, spherically symmetric fluid. This talk will highlight some of the tools used to prove the existence and regularity of solutions to

$$\begin{cases} u_t = d\Delta u + \frac{f(x)}{u^p v^q}, \\ v_t = D\Delta v + \frac{g(x)}{u^r v^s}, \end{cases}$$

and

$$\begin{cases} u_t = d\Delta u + \frac{f_1(x)}{u^p} + \frac{f_2(x)}{v^q}, \\ v_t = D\Delta v + \frac{f_3(x)}{u^r} + \frac{f_4(x)}{v^s}. \end{cases}$$

After perturbing the system, a functional method is used to obtain uniform $L^k(\Omega)$ bounds for the regularized solution $(u_\varepsilon, v_\varepsilon)$. These bounds allow us to prove the existence of a positive, weak solution (u, v), and in some cases the solution can be shown to be classical. Further, the solution is shown to exist for all time, and in some cases remains bounded for all time. These results are generalizations of previous work done with Dr. Chen and Dr. Xu featuring similar nonlinearities.