
Singularities and Phase transitions in Nonlinear PDE's

Singularités et transitions de phases dans les équations aux dérivées partielles non linéaires

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ROHIT JAIN, McGill University

The Two-Penalty Boundary Obstacle Problem

In this talk we study a two-penalty boundary obstacle problem of interest in surface diffusion, fluid dynamics and semi-permeable membrane theory. In particular, we discuss existence and uniqueness theory, optimal regularity of the solution, and structural properties of the free boundary. This is joint work with Donatella Danielli and Thomas Backing.

MAXIME LABORDE, McGill University

An unbalanced Optimal Transport splitting scheme for general advection-reaction-diffusion problems

In this talk I will present a joint work with T.O. Gallouët and L. Monsaingeon. We show that unbalanced optimal transport provides a convenient framework to handle reaction and diffusion processes in a unified metric framework. We use a constructive method, alternating minimizing movements for the Wasserstein distance and for the Fisher-Rao distance, and prove existence of weak solutions for very degenerate reaction-diffusion-advection equations of the form $\partial_t \rho - \operatorname{div}(\rho \nabla(V + p)) = \rho \Phi(p, x)$ where the potential drift V and the reaction term $\Phi(\cdot, x)$ are given. The diffusion nonlinearity can be of the Porous-Medium type $p = g_m(\rho) = \frac{m}{m-1} \rho^{m-1}$, and in the limit $m \rightarrow +\infty$ the diffusion degenerates in a Hele-Shaw type problem with hard constraint $\rho \leq 1$ and the maximal monotone graph relation: $p \in g_\infty(\rho) = 0$ if $0 \leq \rho < 1$ and $p \in g_\infty(\rho) = [0, +\infty$ if $\rho = 1$. Moreover, some numerical simulations will be presented.

XIN YANG LU, Lakehead University

Nonlocal evolution equations from epitaxy and liquid crystals

Epitaxy is a process in which a thin film is grown above a much thicker substrate. Even in the simplest case, with no deposition, and purely elastic interactions, such growth leads to a nonuniform film thickness since the film and the substrate can have different rigidity constants. The resulting system is thus an energy driven one, but quite irregular. Moreover, since the underlying mechanism involve strongly nonlocal interactions, such energies generally contain some nonlocal terms. Similarly, the evolution of nematic liquid crystals, systems is modeled by a highly complex energy driven system. In this talk I will present some recent results about the regularity of solutions to several equations arising from nematic liquid crystals and epitaxy.

GUANYING PENG, University of Arizona

On the first critical field of a 3D anisotropic superconductivity model

We analyze the Lawrence-Doniach model for layered superconductors occupying a bounded generalized cylinder, $\Omega \times (0, L)$, in \mathbb{R}^3 , where the cross-section Ω is a bounded simply connected domain in \mathbb{R}^2 . For an applied magnetic field $\vec{H}_{ex} = h_{ex} \vec{e}_3$ that is perpendicular to the layers, there are two critical values for h_{ex} at which the superconductor has phase transitions. In this talk, we will present some recent work on characterizations of the first critical field at which the magnetic field first penetrates the superconductor to create defects in the material. Such characterizations are achieved by analyzing a mean field model that is the Gamma-limit of the Lawrence-Doniach model in appropriate regimes.

SCOTT RODNEY, Cape Breton University

Poincaré Inequalities and Neumann problems for the p -Laplacian

Reporting on joint work with D. Cruz-Urbe and E. Rosta, I will discuss an equivalence between the validity of a matrix weighted Poincaré inequality and the existence of a regular weak solution to a Neumann problem for a degenerate p -Laplacian. The Poincaré inequalities discussed are formulated in the context of degenerate Sobolev spaces defined in terms of a quadratic form, and the associated matrix is the source of the degeneracy in the p -Laplacian.

HOLGER TEISMANN, Acadia University

Minimal time for the approximate bilinear control of Schrödinger equations

The topic of this talk is in the area of quantum control to which we give a very brief introduction before discussing the result announced in the title. The purpose of the research is to delineate what can be accomplished by the application of classical fields; in particular, whether control can be achieved in arbitrarily small time. Departing from a result by Boscain et al., which ensures approximate controllability in (potentially) large time, we present a large class of quantum systems for which we show that approximate control fails in small time, and that this failure is (essentially) independent of the initial state.

Joint work with K. Beauchard (ENS Rennes) and J.-M. Coron (Paris 6).