Partial Differential Equations and Variational Problems
Équations aux dérivées partielles et problèmes variationnels
(Org: **Mohammad El Smaily** (University of New Brunswick))

ZACHARY BRADSHAW, University of Arkansas
Eventual and far-field regularity of some infinite energy solutions to the Navier-Stokes equations

Leray weak solutions are known to exhibit both eventual and far-field regularity. These properties have not been investigated for local Leray solutions, a useful generalized solution class introduced by Lemarie-Rieusset. We establish several eventual and far-field regularity criteria depending on properties of the initial data. These have interesting implications for local Leray solutions with initial data in a wide variety of familiar spaces, e.g. the critical Lebesgue space and the critical, end-point Lorentz space. As an application, we present new regularity criteria for some discretely self-similar solutions to the Navier-Stokes equations.

GRAHAM COX, Memorial University
Nodal deficiency, spectral flow, and the Dirichlet-to-Neumann map

Courant's nodal domain theorem provides a natural generalization of Sturm–Liouville theory to higher dimensions; however, the result is in general not sharp. It was recently shown that the nodal deficiency of an eigenfunction is encoded in the spectrum of the Dirichlet-to-Neumann operators for the eigenfunction's positive and negative nodal domains. While originally derived using symplectic methods, this result can also be understood through the spectral flow for a family of boundary conditions imposed on the nodal set. In this talk I will describe this flow for a Schrödinger operator with separable potential on a rectangular domain, and describe a mechanism by which low energy eigenfunctions do or do not contribute to the nodal deficiency. Operators on non-rectangular domains and quantum graphs will also be discussed.

This talk represents joint work with Gregory Berkolaiko (Texas A&M) and Jeremy Marzuola (UNC Chapel Hill).

THEODORE KOLOKOLNIKOV, Dalhousie University
Pattern formation in the presence of space-dependent parameters

In first part of the talk, we consider a standard reaction-diffusion system (the Schnakenberg model) that generates localized spike patterns. Our goal is to characterize the distribution of spikes and their heights in the limit of many spikes, in the presence of spatially-dependent feed rate A(x). This leads to an unusual nonlocal problem for spike locations and their heights. A key ingredient in the solution of the reduced problem is the Euler-Maclaurin formula. In the second part of the talk, we look distribution of vortices in Bose-Einstein Condensates with a parabolic trap. We find novel equation of motion for vortex centers, and use these equations to derive vortex density as well as the theoretical maximum number of vortices that can fit within the vortex lattice for a given rotation rate. Our results improve on known results in the literature.

NAM QUANG LE, Indiana University, Bloomington
Holder regularity of the 2D dual semigeostrophic equations

The semigeostrophic equations are a simple model used in meteorology to describe large scale atmospheric flows. They can be derived from the three-dimensional incompressible Euler equations, with Boussinesq and hydrostatic approximations, subject to a strong Coriolis force. Since for large scale atmospheric flows the Coriolis force dominates the advection term, the flow is mostly bi-dimensional. In this talk, we discuss the Holder regularity of time derivative of solutions to the dual semigeostrophic equations in two dimensions when the initial potential density is bounded away from zero and infinity. Our main tool is an interior Holder estimate in two dimensions for an inhomogeneous linearized Monge-Ampere equation with right hand side being the divergence of a bounded vector field.

SAIKAT MAZUMDAR, University of British Columbia
Blow-up Analysis for a Nonlinear Elliptic Equation with Critical growth and Hardy weight.

In this talk we will study the asymptotic behaviour of a sequence of solutions to a family of elliptic PDEs with Hardy weight and Sobolev critical growth. The blow-up analysis will give us conditions under which compactness holds for this sequence and this in turn will help establish the existence of solutions. The location of the singularity, be it in the interior of the domain or on its boundary, affects the analytical properties of the equation and makes the two situations quite different. When the singularity is in the interior then a lower order perturbation suffices for high dimensions, while the curvature of the boundary plays a crucial role if the singularity is on the boundary. This is a joint work with Nassif Ghoussoub(UBC) and Frederic Robert(Universite de Lorraine).

SCOTT RODNEY, Cape Breton University
Global Sobolev Inequalities and Degenerate p-Laplacians

Reporting on joint work with D. Cruz-Uribe and E. Rosta, I discuss a local-global result for matrix weighted Sobolev inequalities using a PDE approach. Given a non-negative definite $n \times n$ matrix function $Q = Q(x)$ in a domain Ω of $\mathbb{R}^n$, our main result is achieved through a regularity analysis for a one parameter family of matrix weighted p-Laplacians ($p > 1$) of the form

$$X_{p,\tau}u = \mathrm{div}\Big(\big|\sqrt{Q}\nabla u\big|^{p-2}Q\nabla u\Big) - \tau|u|^{p-2}u$$

for $\tau \in (0, 1)$.

TUOC VAN PHAN, University of Tennessee
Regularity estimates for BMO-weak solutions of quasilinear elliptic equations with inhomogeneous boundary conditions

This talk is about the regularity estimates in Lebesgue spaces for gradients of weak solutions of a class of general quasilinear equations of p-Laplacian type in bounded domains with inhomogeneous conormal boundary conditions. In the considered class of equations, the principals are vector field functions measurable x-variable, and nonlinearly depending on both solution and its gradient. This class of equations consists of the well-known class of degenerate p-Laplace equations for $p > 1$. Under some sufficient conditions, we establish local interior, local boundary, and global $W^{1,q}$-regularity estimates for weak solutions with $q > p$, assuming that the weak solutions are in the John-Nirenberg BMO space. The obtained results improve available results because they do not require the boundedness or continuity assumptions on solutions. The results also unify and cover known results for equations in which the principals are vector field function of only x-variable and the gradient of solution variable. More than that, we also give a method to treat non-homogeneous boundary value problems directly without using any form of translations that is sometimes complicated due to the nonlinearities.

XINWEI YU, University of Alberta
A New Prodi-Serrin Type Regularity Criterion in Velocity Directions

We prove that a Leray-Hopf weak solution for the 3D incompressible Navier-Stokes equations would stay regular if a Prodi-Serrin type integrability condition in Lorentz spaces is satisfied by $div(u/|u|)$, where u is the velocity function. This is joint work with Mr. Benjamin Pineau.