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*Factorization tests arising from counting modular forms and automorphic representations*

A theorem of Gekeler compares the number of non-isomorphic automorphic representations associated with the space of cusp forms of weight  $k$  on  $\Gamma_0(N)$  to a simpler function of  $k$  and  $N$ , showing that the two are equal whenever  $N$  is squarefree. We prove the converse of this theorem (with one small exception), thus providing a characterization of squarefree integers. We also establish a similar characterization of prime numbers in terms of the number of Hecke newforms of weight  $k$  on  $\Gamma_0(N)$ .

It follows that a hypothetical fast algorithm for computing the number of such automorphic representations for even a single weight  $k$  would yield a fast test for whether  $N$  is squarefree. We also show how to probabilistically obtain the complete factorization of the squarefull part of  $N$  from the number of such automorphic representations for two different weights. If in addition we have the number of such Hecke newforms for one more weight  $k$ , then we show how to probabilistically factor  $N$  entirely. All of these computations could be performed quickly in practice, given the number(s) of automorphic representations and modular forms as input. (joint work with Miao Gu)