EGBERT RIJKE, Carnegie Mellon University
The join construction

In homotopy type theory we can define the fiberwise join of maps as a binary operation on maps with a common codomain, by first taking the homotopy pullback, and then the homotopy pushout. This operation is commutative, associative, and the unique map from the empty type into the common codomain is a neutral element. Moreover, the idempotents of the join of maps are precisely the embeddings.

We define the image of a map $f : A \to X$ as the colimit of the finite join powers of f. This construction of the image is called the join construction. The join powers therefore provide approximations of the image inclusion. These approximations can be of interest themselves: for instance the projective spaces appear as such.

Furthermore, we show that if A is essentially small (with respect to a universe $\mathcal{U}$), and X is locally small (w.r.t. $\mathcal{U}$), then the image of f is again essentially small. This observation helps us to obtain a type theoretic version of the replacement axiom, and we use it to show that quotients of small types are again small.

Using the idea of (higher) quotients we are then able to show that for any reflective subuniverse (a subuniverse of which the inclusion functor has a left adjoint), the subuniverse of separated objects (of which the identity type is in the original reflective subuniverse) is again a reflective subuniverse.