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Poincaré inequalities for Sobolev spaces with matrix valued weights and applications

For bounded domains of $\mathbb{R}^n$, we prove that the L^p-norm of a regular function with compact support is controlled by weighted L^p-norms of its gradient, where the weight belongs to a class of symmetric non-negative definite matrix valued functions. The class of weights is defined by regularity assumptions and structural conditions on the degeneracy set S, where the determinant vanishes. In particular, S is assumed to be a sufficiently regular compact submanifold of $\mathbb{R}^n$ (with or without boundary) and the matrix weight A is assumed to have rank at least one when restricted to the normal bundle of the degeneracy set S. As an auxiliary result of independendent interest, we also prove a regularity result for the distance function from a compact submanifold with boundary in $\mathbb{R}^n$. This generalization of the classical Poincaré inequality can be applied to develop a robust theory of first order L^p-based Sobolev spaces with matrix valued weight A. The Poincaré inequality and these Sobolev spaces can then be applied to produce various results on existence, uniqueness and qualitative properties of weak solutions to boundary value problems for classes of degenerate elliptic, degenerate parabolic and degenerate hyperbolic PDEs of second order written in divergence form. These results are joint work with K.R. Payne (Università degli Studi di Milano) and F. Punzo (Politecnico di Milano).