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Global solutions of a focusing energy-critical heat equation

We study the focusing energy-critical nonlinear heat equation $u_t - \Delta u - |u|^2 u = 0$, in \mathbb{R}^4 . We prove that solutions emanating from initial data with energy and kinetic energy below those of the stationary solution are global and decay to zero. We show that global solutions dissipate to zero building on a refined small data theory and L^2 -dissipation. To rule out the possibility of finite-time blow-up we argue by resorting to the “concentration-compactness plus rigidity” approach of Kenig and Merle for dispersive equations. We exploit the dissipation but our proof does not rely on maximum/comparison principles. The above result extends to all dimensions $d \geq 3$. This is joint work with Stephen Gustafson.