Geometric Methods in Mechanics and Control with Applications
Méthodes géométriques en mécanique et en contrôle avec applications
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PAULA BALSEIRO, Universidade Federal Fluminense, Brazil
Nonholonomic systems and the hamiltonization problem

In this talk we will discuss geometric features of nonholonomic systems and their behavior after a reduction by a group of symmetries.

In particular, we will show how the failure of the Jacobi identity is modified after a reduction by symmetries and also by considering 'gauge related brackets'. We will present some concrete examples where Poisson and twisted Poisson brackets appear in the description of the reduced dynamics. In these cases, we will also discuss the role of conserved quantities.

LARRY BATES, University of Calgary
An extension of the Dirac theory of constraints

Abstract

Constructions introduced by Dirac for singular Lagrangians are extended and reinterpreted to cover cases when kernel distributions are either nonintegrable or of nonconstant rank, and constraint sets need not be closed.

RICHARD CUSHMAN, University of Calgary
The spherical pendulum

This talk will describe the geometry of the energy momentum mapping of the classical spherical pendulum.

FRANCOIS GAY-BALMAZ, Ecole Normale Superieure de Paris
Multisymplectic integrators for nonsmooth mechanics

We present a class of multisymplectic variational integrators for nonsmooth continuum mechanics. Typical problems are the impact of an elastic body on a rigid surface or the collision of two elastic bodies. The integrators are obtained by combining, at the continuous and discrete levels, the variational multisymplectic formulation of nonsmooth continuum mechanics with the generalized Lagrange multiplier approach for optimization problems with nonsmooth constraints. These integrators verify a spacetime multisymplectic formula that generalizes the symplectic property of time integrators. In addition, they preserve the energy during the impact. In presence of symmetry, a discrete version of Noether's theorem is verified. All these properties are inherited from the variational character of the integrator. Numerical illustrations are presented.

MARK GOTAY,

MELVIN LEOK, University of California, San Diego
Interpolation on Symmetric Spaces via the Generalized Polar Decomposition

We construct interpolation operators for functions taking values in a symmetric space — a smooth manifold with an inversion symmetry about every point. Key to our construction is the observation that every symmetric space can be realized as a homogeneous space whose cosets have canonical representatives by virtue of the generalized polar decomposition — a generalization of the well-known factorization of a real nonsingular matrix into the product of a symmetric positive-definite matrix times an orthogonal matrix. By interpolating these canonical coset representatives, we derive a family of structure-preserving interpolation operators for symmetric space-valued functions. As applications, we construct interpolation operators for the space of Lorentzian metrics, the space of symmetric positive-definite matrices, and the Grassmannian. In the case of Lorentzian metrics, our interpolation operators provide a family of finite elements for numerical relativity that are frame-invariant and have signature which is guaranteed to be Lorentzian pointwise. We illustrate their potential utility by interpolating the Schwarzschild metric numerically. This is joint work with Evan Gawlik.

ANDREW LEWIS, Queen's University
Tautological Control Systems

A framework for geometric control theory is presented. The objectives of the framework are: (1) to be independent of any parameterisation of controls, cf. coordinate-invariance in differential geometry; (2) to be able to seamlessly incorporate real analytic system models; (3) to permit general control sets; (4) to incorporate locally defined data. The putting in place of these objectives is intended to ensure that models do not obstruct any understanding of their fundamental structural properties, e.g., controllability, stabilisability, optimality. The meaning of each of these objectives is explained, and a framework achieving them is presented. The framework requires suitable topologies for spaces of vector fields, particularly real analytic vector fields. The use of topologies allows a unified treatment of different regularity classes: finitely differentiable, Lipschitz, smooth, real analytic. This, per se, provides some interesting results concerning the flows of vector fields depending measurably on time.

GEORGE PATRICK,

VAKHTANG PUTKARADZE, University of Alberta
Geometric approach to the dynamics of tubes conveying fluid

We derive a fully three-dimensional, geometrically exact theory for flexible tubes conveying fluid. The theory also incorporates the change of the cross-section available to the fluid motion during the dynamics, sometimes called collapsible tubes. Our approach is based on the symmetry-reduced, exact geometric description for elastic rods, coupled with the fluid transport and subject to the volume conservation constraint for the fluid. Using these methods, we derive the fully three dimensional equations of motion. We then proceed to the linear stability analysis and show that our theory introduces important corrections to previously derived results, both in the consistency at all wavelength and in the effects arising from the dynamical change of the cross-section. We also derive and analyze several analytical, fullynonlinear solutions of traveling wave type in two dimensions. Finally, time permitting, we present the variational scheme for discretization of dynamics. This research has been supported by NSERC and the University ofAlberta.

STUART ROGERS, University of Alberta
Optimal Control of a Nonholonomic Mechanical System

This talk investigates the optimal control of a mechanical system with nonholonomic constraints. Suslov's problem is an algebraically simple and classical example of a nonholonomic mechanical system. This mechanical system considers the motion of a rigid body rotating about its center of mass subject to the constraint $\Omega(t) \cdot \xi(t) = 0$, where $\Omega(t)$ denotes the rigid body's angular velocity and $\xi(t)$ is a prescribed time-varying vector, both expressed in the rigid body's body frame. First, the pure equations of motion of this nonholonomic mechanical system are derived. Next, letting $\xi(t)$ serve as the control, the optimal control equations of motion are derived that obey the pure equations of motion, satisfy prescribed initial and terminal boundary conditions, and minimize the time integral of a prescribed cost function $C(t, \Omega(t), \dot{\Omega}(t), \xi(t), \dot{\xi}(t))$. Finally, numerical solutions of the optimal control equations are presented.

JEDRZEJ SNIATYCKI, Universities of Victoria and Calgary
Quantum Spherical Pendulum

Spherical pendulum is an example of a completely integrable system with globally defined continuous action functions. Since the derivatives of actions are discontinuous, we do not have global angle variables (monodromy). Nevertheless, the quantum states of the spherical pendulum, defined by the Bohr-Sommerfeld conditions, form a 2-dimensional lattice with boundary. Operators of shifting along the generators of the lattice are well defined and lead to a full quantum theory of the spherical pendulum.

HIROAKI YOSHIMURA, Waseda University
Discrete Dirac Structures and Nonholonomic Integrators for Lagrange-Dirac Systems

Much effort has been devoted to developing numerical integrators for nonholonomic mechanical systems. There is no doubt that the variational integrator which preserves a discrete structure of system dynamics is an essential tool for the numerical analysis of such nonholonomic systems. In recent years, the continuous and discrete settings of Dirac structures and associated Lagrangian and Hamiltonian dynamical systems have been developed for modeling and analysis of nonholonomic mechanical systems. In this talk, we propose a discrete Dirac structure and associated Lagrange-Dirac dynamical systems which is slightly different from previous works. In particular, we will show the property of preserving the discrete Dirac structure. Then we show the link between the discrete Dirac dynamical systems and the discrete Lagrange-d'Alembert-Pontryagin variational structures. We illustrate our theory by some illustrative examples together with numerical computations. This research is a joint work with Linyu Peng.

DMITRY ZENKOV, North Carolina State University
The Helmholtz Conditions and the Method of Controlled Lagrangians

The method of controlled Lagrangians is a technique for deriving stabilizing feedback controls for nonlinear mechanical systems. It relies on constructing a Lagrangian which describes the feedback controlled dynamics. The talk will review the connections between the classical inverse problem of the calculus of variations and the method of controlled Lagrangians. This is a joint work with Anthony Bloch and Demeter Krupka