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Linked systems of symmetric designs and real mutually unbiased bases

Let Γ be a finite undirected graph with vertex set X partitioned into w subsets each of size v:

$$X = X_1 \dot{\cup} X_2 \dot{\cup} \cdots \dot{\cup} X_w \ .$$

We say that Γ is a *linked system of symmetric* (v,k,λ) *designs with* w *fibres* if Γ satisfies the following three properties:
• no edge of Γ has both ends in the same fibre X_i;
• the subgraph of Γ induced between any two distinct fibres X_i and X_j is the incidence graph of some symmetric (v,k,λ) design;
• for any three distinct indices i,j,k from $\{1,\ldots,w\}$ and for any $a \in X_i$ and any $b \in X_j$ the number of common neighbors of a and b lying in X_k depends only on whether or not (a,b) is an edge of Γ and not on the choice of a and b or on the choice of i,j,k.

A set of w *mutually unbiased bases* in $\mathbb{R}^d$ ("w real MUBs") is a collection of orthogonal bases $\mathcal{B}_1,\ldots,\mathcal{B}_w$ for $\mathbb{R}^d$ enjoying the property that $|\mathbf{x}\cdot\mathbf{y}|$ is constant whenever $\mathbf{x}$ and $\mathbf{y}$ are chosen from distinct bases $\mathcal{B}_i$ and $\mathcal{B}_j$ from our collection.

In this talk we determine when a linked system of symmetric designs can be converted into a set of real MUBs and give partial results in the reverse direction. The talk is based, in part, on joint work with my student Brian Kodalen.