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A Zero Density Result for the Riemann Zeta Function

Let $N(\sigma, T)$ denote the number of nontrivial zeros of the Riemann zeta function with real part greater than σ and imaginary part between 0 and T . We provide explicit upper bounds for $N(\sigma, T)$ commonly referred to as a zero density result. In 1940, Ingham showed the following asymptotic result

$$N(\sigma, T) = O(T^{\frac{3(1-\sigma)}{2-\sigma}} \log^5 T).$$

Ramaré recently proved an explicit version of this estimate:

$$N(\sigma, T) \leq 4.9(3T)^{\frac{8}{3}(1-\sigma)} \log^{5-2\sigma}(T) + 51.5 \log^2 T,$$

for $\sigma \geq 0.52$ and $T \geq 3.061 \cdot 10^{10}$. We discuss a generalization of the method used in these two results which yields an explicit bound of a similar shape while also improving the constants. Furthermore, we present the effect of these improvements on explicit estimates for the prime counting function $\psi(x)$. This is joint work with Habiba Kadiri and Nathan Ng.