Partial Differential Equations
Équations aux dérivées partielles
(Org: **Mostafa Fazly** (Alberta) and/et **Juncheng Wei** (UBC))

WEIWEI AO, ubc
Existence of positive solutions with a prescribed singular set of fractional Yamabe Problem

In this paper, we consider the problem of the existence of positive solutions with prescribed isolated singularities of

$$\begin{cases} (-\Delta)^s u = u^{\frac{n+2s}{n-2s}} \text{ in } R^n \\ u(x) \to \infty \text{ as } x \to \Sigma \end{cases}$$

where Σ is a set of discrete points in R^n. Near each singular point, these solutions are approximated by the Delaunay-type singular solution which has been studied recently by De la Torre, Del Pino, Mar Gonzalez and J.C. Wei. Away from the singular points, these solutions are approximated by the summation of the Green's function. This result is the analogous result for the classical Yamabe problem studied by Mazzeo and Pacard (1999). This is a joint work with De la Torre, Mar Gonzalez and J.C. Wei.

ALEXEI CHEVIAKOV, University of Saskatchewan
Conservation laws of vorticity-type equations

Partial differential equations (PDE) of the form $\operatorname{div}\vec{N} = 0$, $\vec{N}_t + \operatorname{curl}\vec{M} = 0$ involving two vector functions $\vec{N}, \vec{M} \in \mathbb{R}^3$ that depend on t, x, y, z arise as subsets of PDE systems in various models, including the vorticity formulation of viscous and inviscid fluid dynamics, plasma physics (magnetohydrodynamics), and Maxwell's equations. We refer to these equations as "vorticity-type equations".

It is shown that vorticity-type equations have a special structure of a lower-degree (degree two) conservation law in $\mathbb{R}^4(t, x, y, z)$. Moreover, they form an abnormal PDE system, in the sense of possessing an identically vanishing differential consequence. Even though vorticity-type equations are not variational, a result similar to the Noether's second theorem holds: these equations admit an infinite-dimensional family of conservation laws involving an arbitrary function of all variables. Applications of these conservation laws and related results are discussed.

MATT COLES, University of British Columbia
Subcritical Perturbations of Energy Critical NLS

The 3D energy critical focusing nonlinear Schrödinger equation (NLS) admits the so called Aubin-Talenti standing wave solutions. These functions were crucial in Kenig and Merle's 2006 scattering result. We will consider the above equation but whose nonlinearity is perturbed by a small subcritical term. We construct solitary wave solutions as perturbations of the Aubin-Talenti function by means of Lyapunov-Schmidt reduction and resolvent expansion. We further demonstrate these solutions to be ground states and use them to achieve a scattering result in the spirit of Kenig and Merle.

STEPHEN GUSTAFSON, University of British Columbia
Stability of periodic waves of 1D nonlinear Schrödinger equations

The cubic focusing and defocusing Schrödinger equations in one dimension admit periodic wave solutions given by snoidal, cnoidal, and dnoidal Jacobi elliptic functions. We examine the stability of these solutions, and among other things prove the spectral stability of the cnoidal waves (in a certain parameter range) with respect to same-period perturbations. This is done via variational and spectral analysis, while as much as possible avoiding the use of complete integrability. This is joint work with Stefan Le Coz and Tai-Peng Tsai.

THEODORE KOLOKOLNIKOV, Dalhousie
Spike distribution density in a reaction-diffusion system with spatial dependence

We consider a standard reaction-diffusion system (the Schnakenberg model) that generates localized spike patterns. Our goal is to characterize the distribution of spikes and their heights in the limit of many spikes, in the presence of spatially-dependent feed rate $A(x)$. This leads to an unusual nonlocal problem for spike locations and their heights. A key feature of the resulting nonlocal problem is that it is necessary to estimate the difference between the continuum limit and the discrete algebraic system to derive the effective spike density. In a certain limit, we find that the effective spike density scales like $A^{2/3}(x)$ whereas the spike heights scale like $A^{1/3}(x)$. In another limit, we derive instability thresholds for when N spikes become unstable.

M. NIKSIRAT, University of Toronto
On the existence of periodic solutions to nonlinear evolution equations

Let $\Omega \subset \mathbb{R}^n$ be an open bounded set and $T > 0$. We are concern with the existence of periodic solutions to the evolution equation

$$\begin{cases} \partial_t u + f(t, x, D^{\leq 2}u) = 0 \\ u(0) = u(T) \end{cases},$$

where $x \in \Omega$ and f is T-periodic with respect to t and is uniformly elliptic. This means

$$f(t + T, x, D^{\leq 2}u) = f(t, x, D^{\leq 2}u),$$

and

$$\sum_{|\alpha|=2} f_\alpha(t, x, D^{\leq 2}u)\xi^\alpha \geqq \theta|\xi|^2$$

for some $\theta > 0$ and any $\xi \in \mathbb{R}^n - \{0\}$. Letting $u : [0, T] \to X$ where $X = H^{2+n_0}(\Omega) \cap H_0^1(\Omega)$ for $n_0 = \left[\frac{n}{2}\right] + 1$, we generalize the Skrypnik's degree for fully nonlinear elliptic maps of the form $f(x, D^{\leq 2}u)$ to one-parameter family of maps of the form $f(t, x, D^{\leq 2}u)$ and then use the Browder's degree for maps of the form $A = T + \varphi$, where T is a maximal monotone map and φ is a $(S)_+$ map to establish conditions for the existence of a periodic solution to fully nonlinear evolution equations.

CHUNHUA OU, Memorial University
Periodic steady-state solutions to the liquid film flowing model: existence and stability

In this talk, we investigate the dynamics of a liquid film flowing over a periodic wavy wall. This study is based on a long-wave model that is valid at near-critical Reynolds number. For the periodic wall surface, we construct an iteration scheme in terms of an integral form of the original steady-state problem. The uniform convergence of the scheme is proved so that we can derive the existence, uniqueness as well as the asymptotic formula of the periodic solutions. These results can also be obtained by the method of abstract contraction mapping in a particular functional space. Using the Floquet-Bloch theory, we establish several analytic results on the stability/instability of the periodic steady-state solutions in a weighted functional space.

BRENDAN PASS, University of Alberta
Multi - to one - dimensional transportation

I will discuss joint work with Pierre-Andre Chiappori and Robert McCann on the Monge-Kantorovich problem of transporting a probability measure on $\mathbf{R}^n$ to another on the real line. We introduce a nestededness criterion relating the cost to the marginals, under which it is possible to solve this problem uniquely (and essentially explicitly), by constructing an optimal map one level set at a time. I plan to discuss examples for which the nestedness condition holds, as well as some for which it fails; some of these examples arise from a matching problem in economics which originally motivated our work. If time permits, I will also briefly discuss how level set dynamics can be used to develop a local regularity theory in the nested case.

CRISTIAN RIOS, University of Calgary
Harnack's estimates for fractional powers of non-symmetric second order operators

We build on the Caffarelli-Silvestre treatment of fractional powers of elliptic operators and extend it to non-symmetric operators on Banach spaces. To illustrate the reach of this more general extension theorem we obtain a-priory boundedness of solutions and Harnack's estimates for solutions of fractional powers of weighted elliptic operators, subelliptic operators, and operators in nondivergence form.

DIMITRIOS ROXANAS, University of British Columbia
Global solutions of a focusing energy-critical heat equation

We study the focusing energy-critical nonlinear heat equation $u_t - \Delta u - |u|^2 u = 0$, in $\mathbb{R}^4$. We prove that solutions emanating from initial data with energy and kinetic energy below those of the stationary solution are global and decay to zero. We show that global solutions dissipate to zero building on a refined small data theory and L^2-dissipation. To rule out the possibility of finite-time blow-up we argue by resorting to the "concentration-compactness plus rigidity" approach of Kenig and Merle for dispersive equations. We exploit the dissipation but our proof does not rely on maximum/comparison principles. The above result extends to all dimensions $d \geq 3$. This is joint work with Stephen Gustafson.

ERIC SAWYER, McMaster
Local boundedness and maximum principle for rough infinitely degenerate elliptic operators

We prove local boundedness for a class of rough degenerate operators, which is sharp in three dimensions, and a maximum principle for a larger family of operators. This is joint work with Luda Korobenko, Cristian Rios and Ruipeng Shen.

IHSAN TOPALOGLU, McMaster University
Small volume fraction limit of a nonlocal isoperimetric problem with confinement

The nonlocal isoperimetric problem that I will consider in this talk arises as the sharp interface limit of Ohta-Kawasaki functionals introduced to model microphase separation of diblock copolymers. In our problem there is an additional term which penalizes one phase hence forces the other block copolymer phase into a confinement region. Using Γ-convergence we will identify the first- and second-order effective energies in the asymptotic limit of small volume fraction and strong confinement. Depending on the choice of penalization we will show that the second-order limit of these energies will be given by attractive-repulsive nonlocal interaction energies of weighted Dirac-delta functions corresponding to the concentration of mass into point particles. This is a joint work with S. Alama, L. Bronsard and R. Choksi.

JEROME VETOIS, McGill University
Clustering solutions for smooth perturbations of a critical elliptic equation in low dimensions

I will discuss the question of existence of positive solutions with multiple clustering peaks for smooth perturbations of a critical elliptic equation on a closed manifold. I will present an existence result of such solutions for the Lin-Ni equation in dimensions four and five on a manifold with negative scalar curvature. This is a joint work with Pierre-Damien Thizy (University of Cergy-Pontoise).

ERIC WOOLGAR, University of Alberta
Ricci flow of partially even metrics

Both the general variational theory and the Ricci flow theory of compact Riemannian manifolds have been widely studied. The noncompact case, however, raises many open issues. A nice place to start is with the conformally compactifiable, asymptotically hyperbolic metrics. These have enough structure at infinity to permit the definition of renormalized curvature integrals, which

may then admit a variational theory and may also provide possible monotonic quantities for Ricci flow. We consider the "partially even" metrics. These are conformally compactifiable and asymptotically hyperbolic, and admit a Fefferman-Graham expansion whose first few terms are even. If such a metric also obeys a certain trace condition, it is volume renormalizable. We show that normalized Ricci flow preserves the partially even and volume renormalizable properties, and we obtain a formula for the change in renormalized volume along the flow. This is joint work with Eric Bahuaud and Rafe Mazzeo.

XINWEI YU, University of Alberta

Regularity criteria of 3D Navier-Stokes Equations involving the pressure term

In this talk we will present some new regularity criteria for the 3D Navier-Stokes equations. This new criteria involve combinations of the pressure and the velocity. They improve the classical Prodi-Serrin conditions. This is joint work with Prof. Chuong V. Tran of University of St. Andrews, Scotland.