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Some computational evidence on the heuristics of Guy and Selfridge

Let $s(n)$ denote the sum of the proper divisors of a positive integer n. An aliquot sequence is a sequence of the form $n, s(n), s_2(n) = s(s(n)), s_3(n) = s(s(s(n)))$, and so on. In 2003, Bosma and Kane proved that the geometric mean of $s(2n)/(2n)$ exists and is slightly less than one. Recently, Carl Pomerance demonstrated that the geometric means of $s(s(2n))/s(2n)$ and $s(2n)/(2n)$ for $n > 1$ match. Both of these results give a strong probabilistic evidence that most of the aliquot sequences starting with an even number are bounded. In our work, we show that the geometric means of $s_k(2n)/s_{k-1}(2n)$ for $2n \leq X$ exceed one for $X = 2^{37}$ and $k = 6, 7, 8, 9, 10$ when averaged over all n such that $s_k(2n) > 0$. Moreover, as k increases, the geometric means grow, too. However, as k remains fixed, the geometric means decrease with the growth of X, possibly approaching the geometric mean of $s(2n)/(2n)$. This can be counted as a computational evidence both for and against the heuristics of Guy and Selfridge given in 1976 that most of the aliquot sequences starting with an even number should be unbounded.