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Regularization of the norm of random matrices

We study $n \times n$ matrices A with i.i.d. entries having zero mean and unit variance. If the entries are also subgaussian, then the operator norm $\|A\| \sim O(\sqrt{n})$ with high probability, but without subgaussian assumption the norm can be much larger, in some examples it is up to $O(n)$.

So, we are motivated by the question: what is it in the structure of a heavy-tailed matrix that makes its norm to blow up? We show that with high probability the problem is "local": there is a $\varepsilon n \times \varepsilon n$ sub-matrix A_0 (for any $\varepsilon > 0$, i.e. as small as we want), deletion of which regularizes the norm

$$\|A \setminus A_0\| \leq C(\varepsilon)\sqrt{n}$$

We will also discuss the dependence of the norm constant $C(\varepsilon)$ on size parameter ε (we have it optimal up to a logarithmic factor) and how second moment condition is crucial for any "local" regularization. This is a joint work with Roman Vershynin.