Advances in Lyapunov Functions in Mathematical Biology
Percées dans les fonctions de Lyapunov en biologie mathématique
(Org: **Hongbin Guo** (Ottawa) and/et **Connell McCluskey** (Wilfred Laurier))

YOICHI ENATSU, Tokyo University of Science
Effect of waning immunity on asymptotic behavior of epidemic models

Asymptotic behavior of the positive solutions of epidemic models have been widely studied. Waning immunity of recovered individuals has now been highlighted as an important concept of the modelling for the disease prevalence in the population. In this talk, recent works on the asymptotic stability of equilibria of the model governed by a class of nonlinear delay differential equations, are presented. Incorporating two constant delays that represent latency time and infectious period, a several open problems will also be referred concerning the effect of the loss of immunity on the stability of an endemic equilibrium of the model, via the linearization at the equilibrium when the basic reproduction number is greater than 1.

HONGBIN GUO, University of Ottawa
Global stability of stage-structured epidemic models

Stage-structures from various modelling strategies are integrated into standard epidemic models and the global stability of equilibria is consolidated and studied using the graph-theoretic approach.

TOSHIKAZU KUNIYA, Kobe University
Global stability analysis for an age-structured multi-group SIR epidemic model

In this study, we investigate the global asymptotic stability of equilibria in a multi-group SIR epidemic model with discrete and continuous age structures. For both discrete and continuous cases, we obtain the basic reproduction number R_0 as the spectral radius of the next generation matrix/operator and show that if $R_0 < 1$, then the disease-free equilibrium is globally asymptotically stable. Furthermore, under the assumption that the disease transmission coefficient is independent from the age of infective individuals and the rate of removal from infective class is age-independent, we show that if $R_0 > 1$, then the endemic equilibrium is globally asymptotically stable. In the proof, we use the well-known graph theoretic approach developed by Professors Guo, Li and Shuai (2006) together with an approach of max function.

MICHAEL LI, University of Alberta
Global Stability Problems in Heterogeneous Models for Vector-Borne Diseases

The graph-theoretic approach to the construction of global Lyapunov functions for large-scale dynamical systems, developed by Guo-Li-Shuai, was applied to establish global stability of equilibria in mathematical models of vector-borne diseases in heterogeneous populations. Multi-group and multi-city models for Dengue fever are used as examples.

CONNELL MCCLUSKEY, Wilfrid Laurier University
Lyapunov functionals for delay equations with vector borne transmission

Recent work on modifying a Lyapunov function for an ODE to be a Lyapunov functional for a delay differential equation will be presented. In this talk we study the case where the delay comes from vector borne transmission, similar to the situation studied by Ken Cooke in 1979. We build on recent work, identifying conditions under which this "vector-delay" is harmless; it does not disrupt the Lyapunov calculation, and therefore does not affect the global asymptotic stability.

ZHISHENG SHUAI, University of Central Florida
Applications of the graph-theoretic approach to heterogeneous cholera models

The graph-theoretic approach has become a standard method to construct global Lyapunov functions for large-scale differential equation systems. Appropriate graph/network design and reduction is the key in the successful application of the approach. We illustrate these graph/network techniques using various types of cholera models that incorporate heterogenous structures in the host/pathogen (multi-stage or multi-group) and environment (multi-patch or multi-city).

HAITAO SONG, Shanxi University and Lamps York University
Global dynamics of two heterogeneous SIR models with nonlinear incidence rate and delays

To investigate the effect of heterogeneity on the global dynamics of two SIR epidemic models with general nonlinear incidence rate and infection delays, we formulate a multi-group model corresponding to the heterogeneity in the host population and a multi-stage model corresponding to heterogeneous stages of infection. Under biologically motivated considerations, we establish that the global dynamics for each of the two models are determined completely by the corresponding basic reproduction number: if the basic reproduction number is less than or equal to one, then the disease-free equilibrium is globally asymptotically stable and the disease dies out in all groups or stages; if the basic reproduction number is larger than one, then the disease will persist in all groups or stages, and there is a unique endemic equilibrium which is globally asymptotically stable. Then we conclude that the heterogeneity does not change the global dynamics of the SIR model when the incidence rate is a general nonlinear function. Our results extend a class of previous results and can be applied to the other epidemiological models. The proofs of the main results use Lyapunov functional and graph-theoretic approach. This is a joint work with Weihua Jiang(HIT) and Shengqiang Liu(HIT).

YASUHIRO TAKEUCHI, Aoyama Gakuin University
Maturation delay for the predators can enhance stable coexistence in prey-predator model with Allee effect

We consider a prey-predator model with Allee effects in prey growth and Michaelis-Menten type functional response to describe the grazing pattern. We obtain the conditions for stable and oscillatory coexistence of prey and their specialist predator in case of strong and weak Allee effect in prey growth. Main objective of the present work is to show the stabilizing role of maturation delay in the presence of Allee effect in prey growth. We construct the delayed model by incorporating maturation delay parameter and juvenile predators death rate into the growth equation of predators. Apart from the analytical results for the non-delayed and delayed models, we perform extensive numerical simulations to construct the relevant bifurcation diagrams. We provide the biological implications. We discuss on some problem to construct Lyapunov functionals.

JINLIANG WANG, Heilongjiang University
Dynamics of a PDE viral infection model incorporating cell-to-cell transmission

In this talk, I will concern with the global dynamics of a PDE viral infection model with cell-to-cell transmission and spatial heterogeneity. The basic reproduction number $\Re_0$ is obtained in a variational characterization. It is shown that if $\Re_0 \leq 1$, then the infection-free steady state is globally asymptotically stable, while if $\Re_0 > 1$, then the system is uniformly persistent and the infection steady state is globally asymptotically stable. The proof is based on the construction of the appropriate Lyapunov functionals and usage of the Green's first identity together with the LaSalle's invariance principle. In the case of constant parameters, the basic reproduction number $\Re_0$ is easily calculated and the numerical simulation is performed to verify its threshold property.

YANYU XIAO, University of Cincinnati
Application of Lyapunov Functions to some vector-borne disease models

We will use some lyapunov functions to examine the global dynamics of some vector-borne disease models.