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Maximal determinants, sequence pairs, and cyclotomy

Construction of maximal-determinant matrices using pairs of binary sequences with prescribed autocorrelation properties goes back to the work of Paley and Szekeres in the case of Hadamard matrices, and of Ehlich in the case of matrices of size $n \equiv 2 \pmod{4}$. Ehlich's matrices attain the upper bound of Ehlich and Wojtas, which, however, can only be reached under the condition that $n - 1$ be the sum of two integer squares. In certain cases where this condition is not satisfied, new types of sequence pairs discovered about a decade ago produce matrices with determinant approaching the upper bound. We give necessary conditions for the existence of such sequence pairs and describe some methods that have produced solutions up to $n = 106$. Cyclotomic constructions have been fruitful in the search for sequence pairs that generate Hadamard matrices. We describe recent results, some negative, restricting the types of solutions that can be constructed by cyclotomic methods, and some positive, producing infinite families of solutions.

Joint work with Tomas Rokicki, Adam Vollrath, Yancy Liao.