Analytic number theory and Diophantine equations
Analyse et applications d'équations différentielles utilisant des symétries, les lois de la conservation et l'intégrabilité
(Org: **Michael Bennett** (UBC) and/et **Patrick Ingram** (Colorado State University))

AMIR AKBARY, University of Lethbridge
Lang-Trotter conjecture for two elliptic curves

We propose explicit Euler product representation for the universal conjectural constant in the Lang-Trotter conjecture for two elliptic curves. This is joint work with James Parks (KTH Royal Institute of Technology-Sweden).

SHABNAM AKHTARI, University of Oregon
Many Thue equations have no solutions

I will talk about my recent joint work with Manjul Bhargava. We show that a positive proportion of integral binary cubic forms that locally everywhere represent 1 do not globally represent 1. We order all classes of binary cubic forms by their absolute discriminants. We prove the same result for binary forms of any degree greater than 2, provided that these forms are ordered by the maximum of the absolute values of their coefficients.

NILS BRUIN, Simon Fraser University
Obstructions for primitive solutions to $Ax^3 + By^3 = Cz^2$

It is classical that the only obstructions for homogeneous quadratic equations to have primitive solutions (x, y, z) are local; the Hasse principle holds. The picture for $Ax^p + By^q = Cz^r$ is much more complicated. A result by Beukers shows that if $1/p + 1/q + 1/r > 1$ then the primitive solutions correspond to rational points on finitely many genus 0 curves (subject to certain local conditions). However, as for instance $x^2 + 31y^2 = 5z^3$ shows, obstructions to primitivity are not just local anymore: there is also a class group that can provide obstructions.

For other exponents, obstructions are no longer directly explained by class groups. We will explore some statistics in the case $(p, q, r) = (3, 3, 2)$ This is joint work with Patrick McMahon.

KARL DILCHER, Dalhousie University
Derivatives and fast evaluation of the Witten zeta function

We study analytic properties of the Witten zeta function $\mathcal{W}(r, s, t)$, which is also named after Mordell and Tornheim. In particular, we evaluate the function $\mathcal{W}(s, s, \tau s)$ $(\tau > 0)$ at $s = 0$ and, as our main result, find the derivative of this function at $s = 0$, which turns out to be surprisingly simple. These results were first conjectured using high-precision calculations based on an identity due to Crandall that involves a free parameter and provides an analytic continuation. This identity was also the main tool in the eventual proofs of our results. Finally, we derive special values of a permutation sum and study an alternating analogue of $\mathcal{W}(r, s, t)$. (Joint work with Jon Borwein).

AMY FEAVER, The King's University

ALIA HAMIEH, University of Lethbridge
Value-distribution of logarithmic derivatives of L-functions

In this talk, we describe a method for studying the value-distribution of L-functions based on the Jessen-Wintner theory. This method has been explored recently by Ihara and Matsumoto for the case of logarithmic derivatives of Dirichlet L-functions of prime conductor and by Mourtada and V. K. Murty for the case of logarithmic derivatives of Dirichlet L-functions associated with quadratic characters. We show how one may extend such results to the case of cubic characters. This is a work in progress joint with Amir Akbary.

DIMITRIS KOUKOULOPOULOS, Université de Montréal
Sieve weights and their smoothings

I will discuss moments of partially smoothed, truncated divisor sums of the Möbius function. Such divisor sums appear naturally in the theory of the Selberg sieve and they play a key role in the GPY sieve and its recent improvements due to Maynard and Tao. It turns out that if the truncation is smooth enough, the main contribution to the moments comes from almost primes. However, for rougher truncations, the dominant contribution comes from integers with many prime factors.

Analogous questions can be asked for polynomials over finite fields and for permutations, and in these cases the moments behave rather differently, a rare exception. As we will see, a plausible explanation for this phenomenon is given by studying the analogous sums for Dirichlet characters and obtaining different answers depending on whether or not the character is "exceptional".

This is joint work with Andrew Granville and James Maynard.

MATILDE LALIN, Université de Montréal
Polylogarithms and multizeta values in Mahler measure

The Mahler measure of a multivariable polynomial or rational function P is given by the integral of $\log |P|$ where each of the variables moves on the unit circle and with respect to the Haar measure. We will discuss some results involving formulas of Mahler measure of polynomials and rational functions yielding polylogarithms and multizeta values. We will focus specially in families of an arbitrary number of variables.

PETER CHO HO LAM, Simon Fraser University

ALLYSA LUMLEY, York University
A Zero Density Result for the Riemann Zeta Function

Let $N(\sigma, T)$ denote the number of nontrivial zeros of the Riemann zeta function with real part greater than σ and imaginary part between 0 and T. We provide explicit upper bounds for $N(\sigma, T)$ commonly referred to as a zero density result. In 1940, Ingham showed the following asymptotic result

$$N(\sigma, T) = O(T^{\frac{3(1-\sigma)}{2-\sigma}} \log^5 T).$$

Ramaré recently proved an explicit version of this estimate:

$$N(\sigma, T) \leq 4.9(3T)^{\frac{8}{3}(1-\sigma)} \log^{5-2\sigma}(T) + 51.5 \log^2 T,$$

for $\sigma \geq 0.52$ and $T \geq 3.061 \cdot 10^{10}$. We discuss a generalization of the method used in these two results which yields an explicit bound of a similar shape while also improving the constants. Furthermore, we present the effect of these improvements on explicit estimates for the prime counting function $\psi(x)$. This is joint work with Habiba Kadiri and Nathan Ng.

GREG MARTIN, UBC

MYRTO MAVRAKI, University of British Columbia
Simultaneous torsion points in a Weierstrass family of elliptic curves

In 2010, Masser and Zannier proved that there are at most finitely many complex numbers t, not equaling 0 or 1, such that the two points on the Legendre elliptic curve $y^2 = x(x-1)(x-t)$ with x-coordinates 2 and 3 are simultaneously torsion. Recently, Stoll proved that there is in fact no such t, and it is his result that inspires our work. In this talk we will focus on the Weierstrass family of elliptic curves $E_t : y^2 = x^3 + t$, and show that in many instances there is no parameter t such that the points $(a, *)$ and $(b, *)$ are simultaneously torsion in E_t. In contrast to the original approach of Masser and Zannier, our approach is dynamical. We focus on studying whether a and b are simultaneously preperiodic for a Lattès map.

JAMES PARKS, KTH Royal Institute of Technology
Low-lying zeros of quadratic Dirichlet L-functions

In this talk we study the 1-level density of low-lying zeros of Dirichlet L-functions attached to real primitive characters of conductor at most X. We obtain an asymptotic expansion of this quantity with lower order terms in descending powers of $\log X$. We show that this is valid under GRH when the support of the Fourier Transform of the implied even test function ϕ is contained in $(-2, 2)$. We also uncover a phase transition when the supremum of the support of $\widehat{\phi}$ reaches 1, where a new lower order term appears. This is joint work with Daniel Fiorilli and Anders Södergren.

KATHERINE STANGE, University of Colorado Boulder
Arithmetic properties of the Frobenius traces defined by a rational abelian variety

Let A be an abelian variety over the rationals. Under suitable hypotheses, we formulate a conjecture about the asymptotic behaviour of the Frobenius traces $a_{1,p}$ of A reduced modulo varying primes p. This generalizes a well-known conjecture of S. Lang and H. Trotter from 1976 about elliptic curves. We prove upper bounds for the counting function $\#\{p \le x : a_{1,p} = t\}$ and we investigate the normal order of the number of prime factors of $a_{1,p}$. This is joint work with Alina Carmen Cojocaru, Rachel Davis and Alice Silverberg.

GARY WALSH, Universite d'Ottawa
Solving systems of simultaneous Pell equations

We survey recent quantitative results on the subject matter, and then focus on the problem of completely solving certain specific families, generalizing a recent result of Jian Hua Chen. This is joint work with Paul Voutier.