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The Geometric Quantization of a Focus-Focus Singularity

The goal of this project is to find the geometric quantization of a focus-focus system. The focus-focus system is named for the singularity it contains, called the focus-focus singularity. The system is a four-dimensional manifold which can be described in the form of a fibre bundle. The base space of this bundle is a plane, and its ordinary fibres are tori. The focus-focus singularity arises in a singular fibre that takes the form of a pinched torus, where the singularity is the pinch itself. The system exhibits monodromy around the singularity, meaning that similar cycles taken on tori over a cycle around the singularity in the base space are not always consistent. Quantization is a process that takes a symplectic manifold and gives a Hilbert space. There are several different quantization methods, one of which is geometric quantization. Geometric quantization uses the geometric structure of the manifold to build the Hilbert space. Sheaf cohomology is one approach to finding the geometric quantization. The sheaf cohomology can be found using Čech cohomology, which is the method used in this project. To find the Čech cohomology, a covering of the singular region was first constructed. Several versions of this covering were considered, with a goal of finding a covering for which the results can be easily generalized for finer coverings. The covering consists of 11 sets defined over 3 base space regions. The cohomology can be calculated using a parallel transport technique, and partial results have been completed.