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A construction of two different solutions to an elliptic system

Joint work with P.B. Mucha from University of Warsaw. We construct two different solutions to an elliptic system

$$u \cdot \nabla u + (-\Delta)^m u = \lambda F$$

defined on the two dimensional torus. Here $u = (u^1, u^2)$ is sought as a vector function. The operator $(-\Delta)^m$ is elliptic homogenous of order $2m$. It can be viewed as an elliptic regularization of the stationary Burgers 2D system. A motivation to consider the above system comes from an examination of unusual propetries of the linear operator

$$\lambda \sin y \partial_x w + (-\Delta)^m w.$$

Roughly speaking the term with λ effects in a special stabilization of the norms of the operator. We shall underline that the special features of this operator were found firstly via numerical analysis. Our proof is valid for a particular force F and for $\lambda > \lambda_0$, $m > m_0$ sufficiently large. The main steps of the proof concern finite dimension approximation of the system and concentrate on analysis of features of large matrices, which resembles standard numerical analysis. Our analytical results are illustrated by numerical simulations. Experiments are agreed with the conjecture : for small m, in particular for $m = 1$ – for the classical Burgers equation with diffusion, the system does not admit solutions for large λ.

Jacek Cyranka, Piotr Bogusław Mucha, *A construction of two different solutions to an elliptic system*, arXiv:1502.03363.