CHRISTIAN REINHARDT, VU University Amsterdam

Rigorous numerics for nonlinear ODEs using Chebyshev series

In this talk we present a rigorous numerical method to solve initial and boundary value problems for nonlinear ODEs based on Chebyshev series. Our method results in a numerical approximate solution of the ODE together with mathematically rigorous error bounds. The main idea of our proposed approach is to first expand the solution of a given differential equation using its classical Chebyshev series, plug the expansion in the equation and obtain an equivalent infinite dimensional problem of the form $f(x) = 0$ to solve in a Banach space of rapidly decaying Chebyshev coefficients. Via a fixed point argument, we obtain the existence of a genuine solution of $f(x) = 0$ nearby a numerical approximate zero of a finite dimensional projection of f. The Newton-Kantorovich type argument is carried out by using the radii polynomials, which provide an efficient way of constructing a set on which the contraction mapping theorem is applicable. We illustrate the method with examples.