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Local Structure of Singular profiles for a Derivative Nonlinear Schrodinger equation

The Derivative Nonlinear Schrodinger equation $iu_t + u_{xx} + i\left(|u|^2u\right)_x = 0$ (DNLS) is a canonical equation that originally appeared in the study of Hall-MHD equations under a long wavelength scaling. Solutions to the DNLS exist in H^1 locally in time and can be extended for all time provided the initial conditions are small enough: $\|u(t=0)\|_{L^2} < \sqrt{4\pi}$ (Wu 2014). Well posedness for large initial data remains an open problem.

Recent numerical simulations of an L^2-supercritical generalization to the DNLS provide evidence of finite time singularities. Near the singularity, the solution is described by a universal profile that solves a nonlinear elliptic eigenvalue problem depending only on the strength of the nonlinearity. In this work, we use asymptotic analysis to describe the deformation of the profile and the parameters in the limit of criticality. We compare our results to a numerical integration of the problem. This is a joint work with G. Simpson and C. Sulem.