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The instrinsic dynamics of optimal transport

The question of which costs admit unique optimizers in the Monge-Kantorovich problem of optimal transportation between arbitrary probability densities is investigated. For smooth costs and densities on compact manifolds, the only known examples for which the optimal solution is always unique require at least one of the two underlying spaces to be homeomorphic to a sphere. We introduce a (multivalued) dynamics which the transportation cost induces between the target and source space, for which the presence or absence of a sufficiently large set of periodic trajectories plays a role in determining whether or not optimal transport is necessarily unique. This insight allows us to construct smooth costs on a pair of compact manifolds with arbitrary topology, so that the optimal transportation between any pair of probility densities is unique.