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**Recent Advances in the Mathematics of Electromagnetic and Acoustic Imaging**  
**Avancées récentes dans les mathématiques de l'électromagnétisme et de l'imagerie acoustique**  
(Org: **Peter Gibson** and/et **Michael Haslam** (York))

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**PETER GIBSON**, York University

*The non-linear measurement operator for scattering in layered media*

The essential inverse problem in the context of acoustic imaging is to infer physical parameters from measured data, where parameters are mapped to data by the measurement operator. Since the measurement operator is non-linear and poorly understood, a standard approach to the inverse problem is to consider the linearized measurement operator, and view data as arising from perturbation of a given set of parameters. We present recent results concerning layered media that offer an alternative approach. We show that the non-linear measurement operator can be analyzed directly, obviating the need for linearization. The key result is that the measurement operator itself is governed by a PDE with smooth coefficients.

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**MICHAEL HASLAM**, York University

*A High Order Method for Electromagnetic Scattering from Metallic Gratings*

The problem of evaluating the electromagnetic response of a periodic surface to an incident plane wave is of great importance in science and engineering. Applications of the theory exist in several fields of study including solar energy research, optical instrument design, and remote sensing. We discuss the extension of our previous methods to treat the problem of scattering from metallic surfaces with a complex refractive index. The generalization of our methods is not straight-forward, and involves the careful treatment of certain hyper-singular operators which arise in the formulation of the problem in terms of surface integral equations. We demonstrate the rapid convergence of our methods for classically difficult cases in the optical sciences.

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**NICOLAS HOELL**, University of Toronto

*Some Methods in Tomography*

We discuss some advances made in mathematical tomography with applications to various acoustical and nuclear medical imaging modalities. The work rests on combined methods in complex analysis and differential geometry. No background in imaging or tomography is needed.

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**DOUGLAS KURRANT**, University of Calgary

*Microwave breast imaging using patient specific prior information*

Microwave (MW) imaging has been proposed as a complementary method for breast imaging as it is sensitive to the electrical properties of tissues. Radar-based imaging and microwave tomography (MWT) exploit imaging at microwave frequencies. Radar-based imaging uses ultra wideband (UWB) pulses to illuminate the breast and information is extracted from backscattered fields to imply the location of strong scatterers (e.g., tumor). MWT measures the scattered field arising when the breast is illuminated with single frequency time-harmonic waveforms. Iterative methods are applied to the data to solve an inverse scattering problem and reconstruct the spatial distributions of the dielectric properties. High resolution MWT methods use a large number of reconstruction elements to capture details related to spatially fine features within the interior. For typical MW measurement systems, the number of reconstruction elements far exceeds the number of independent data, leading to non-unique solutions. This contributes to the ill-posedness of the problem and manifest as convergence to local minima. Incorporating prior information related to the electrical properties and anatomical structures of the imaged region into MWT inversion algorithms helps to mitigate the problem of ill-posedness. We present a combined radar/MWT method developed for monitoring tumor size changes to assess treatment progress. Radar methods acquire the breast's basic structural information that is used as prior patient specific information for the high resolution MWT algorithm. This basic prior information significantly

improves the quality of the reconstructed images. Joint work with Anastasia Baran (Manitoba), Elise Fear (Calgary), and Joe LoVetri (Manitoba).

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**MARK LYON**, University of New Hampshire

*Accelerated Algorithm for Computing Dielectric Scattering from Scatters Near an Infinite Plane*

We present an algorithm for computing both near-field and far-field scattering emanating from scatters on or near a dielectric plane. Our approach, utilizing both equivalent source points and windowing functions, dramatically reduces the need for expensive evaluations of layered Green's functions. Numerical results determining the accuracy and efficiency of the method will be presented, demonstrating super-algebraic convergence with respect to the size of the windowed region and speeds orders of magnitudes faster than approaches based directly on layered Green's functions. Joint work with Oscar Bruno (Caltech), Carlos Pérez Arancibia (Caltech), and Catalin Turc (NJIT).

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**MOHAMMAD TAVALLA**, York University

*The structure of test functions that determine weighted composition operators*

In the context of analytic functions on the open unit disk, a weighted composition operator is simply a composition operator followed by a multiplication operator. The class of weighted composition operators has an important place in the theory of Banach spaces of analytic functions; for instance, it includes all isometries on  $H_p^2$ . Very recently it was shown that only weighted composition operators preserve the class of outer functions. The present paper considers a particular question motivated by applications: Which smallest possible sets of test functions can be used to identify an unknown weighted composition operator? This stems from a practical problem in signal processing, where one seeks to identify an unknown minimum phase preserving operator on  $L^2(\mathbb{R}_+)$  using test signals. It is shown in the present paper that functions that determine weighted composition operators are directly linked to the classical normal family of schlicht functions. The main result is that a pair of functions  $\{f, g\}$  distinguishes between any two weighted composition operators if and only if there exists a zero-free function  $h$  and a schlicht function  $\sigma$  such that  $\text{span}\{f, g\} = \text{span}\{h\sigma, h\}$ . This solves completely the underlying signal processing problem and brings to light an intriguing geometric object, the manifold of planes of the form  $\text{span}\{h\sigma, h\}$ . As an application of the main result, it is proven that there exist compactly supported pairs in  $L^2(\mathbb{R}_+)$  that can be used to identify minimum phase preserving operators.