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What can we learn from divergent series?

What can we learn from convergent power series? They provide asymptotics close to the origin, they are useful in numerical computations. Moreover, in the complex domain, a convergent series encodes the complete information on the analytic extension of the function which is the sum of the series, including its singularities. But, what about divergent series? For centuries, they have been successfully used in mathematics until the call for rigor banished them from most of mathematics. In this lecture, I will discuss the rehabilitation of divergent series in the 20th century, and how we can rigorously justify their use. I will then move to highlighting the very rich information they can provide on the functions that are their “sums”. The examples presented come from differential equations.