Equitably Coloured BIBDs

A balanced incomplete block design (BIBD) with parameters $v$, $k$ and $\lambda$ consists of a $v$-set $V$ of points together with a set $B$ of $k$-subsets of $V$ called blocks, such that each 2-subset of $V$ is a subset of exactly $\lambda$ blocks of $B$. A colouring of a design $(V,B)$ is a function $f : V \rightarrow C$, where $C = \{c_1, \ldots, c_\ell\}$ is a set of elements called colours. A weak colouring of a design is a colouring $f$ such that $|\{f(x) : x \in B\}| > 1$ for each $B \in B$ (i.e., each block has at least two colours). An equitable colouring is a colouring such that for each block $B \in B$ the number of points of any colour $c_i \in C$ is within 1 of the number of points of any other colour $c_j \in C$ (i.e., $-1 \leq |B \cap C_i| - |B \cap C_j| \leq 1$, where $C_i = \{x \in V : f(x) = c_i\}$ denotes those points of $V$ having colour $c_i$). We determine necessary and sufficient conditions for equitably colourable BIBDs. This is joint work with Robert Luther.