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Ecological models with multiple stable states

Many important questions in ecology and immunology involve dynamical systems with multiple stable states. We present a simple example of intra-guild predation which exhibits a variety of bi- and tri-stabilities. Intra-guild systems can be viewed as a coupling of two predator-prey systems by allowing one predator to additionally prey on the other. The two predators and the resource can coexist in equilibrium, in periodic solutions, and in chaotic solutions. One basic model of intra-guild predation is the system

$$\begin{aligned}R'(t) &= rR(t) \left(1 - \frac{R(t)}{K}\right) - c_1R(t)N(t) - c_2R(t)P(t), \\N'(t) &= e_1c_1R(t - \tau)N(t - \tau) - c_3N(t)P(t) - m_1N(t), \\P'(t) &= e_2c_2R(t)P(t) + e_3c_3N(t)P(t) - m_2P(t).\end{aligned}$$

Here R is a prey species, N and P are predator species, and P additionally preys on N . In contrast with the simple predator-prey model, where delay tends to destabilize the dynamics, we show that the delay in this intra-guild predation model can have both stabilizing and destabilizing effects. In addition we study the appearance and disappearance of an invariant torus in a related model without delay, but with saturating functional and numerical responses:

$$\begin{aligned}R'(t) &= rR(t) \left(1 - \frac{R(t)}{K}\right) - c_1R(t)N(t) - c_2R(t)P(t), \\N'(t) &= e_1c_1R(t)N(t) - \frac{c_3N(t)P(t)}{1 + b_3N(t)} - m_1N(t), \\P'(t) &= \frac{e_2c_2R(t)P(t)}{1 + b_2R(t)} + \frac{e_3c_3N(t)P(t)}{1 + b_3N(t)} - m_2P(t).\end{aligned}$$

Both models illustrate the wide variety of alternate steady-states that are possible in even simple ecological models.