Topics in Approximation Theory
Sujets variés en théorie de l'approximation
(Org: **Kirill Kopotun** and/et **Andriy Prymak** (Manitoba))

ANDRIY BONDARENKO, KNU, Kyiv, Ukraine and NTNU, Trondheim, Norway
On Borsuk's conjecture

In 1933 Karol Borsuk conjectured that every closed set in $\mathbb{R}^n$ of diameter 1 could be partitioned into $n+1$ parts of smaller diameters. The conjecture was disproved in 1993 by Kahn and Kalai. In particular, their construction gives counterexamples for Borsuk's conjecture for $n = 1325$ and for all $n > 2014$. Until recently the best known result was that Borsuk's conjecture is false for all $n \geq 298$.

We will show how to use the Euclidean representation of strongly regular graphs to construct a two-distance set consisting of 416 points on the unit sphere in the dimension 65 which cannot be partitioned into 83 parts of smaller diameter.

ZEEV DITZIAN, University of Alberta
Measure of smoothness on the unit ball

The classic measure of smoothness of functions on a domain D with respect to a norm as applied to the unit ball is not "correct" that is, it does not satisfy matching direct (Jackson) and weak converse inequalities with respect to algebraic polynomial approximation. To deal with the above deficiency F. Dai and Y. Xu introduced a "correct" measure of smoothness on $L_p(B)$.

Here I am introducing a new different measure of smoothness of functions on the unit ball B. This measure beside being "correct" has the added advantages of being rotation invariant (independent of the basis of R^d), being applicable to more norms, and being amenable to more applications. Computability of the new measure was demonstrated and some of the computations show optimality of results.

GERMAN DZYUBENKO, Yu.A.Mitropolskiy International Mathematical Center of NAS of Ukraine
Nearly comonotone approximation of periodic functions

Suppose that a continuous 2π-periodic function f on the real axis changes its monotonicity at points $y_i:\ -\pi \leq y_{2s} < y_{2s-1} < \ldots < y_1 < \pi,\ s \in \mathbb{N}$, on each period. In our resent work, for each $n \geq N$, a trigonometric polynomial P_n of order cn is found such that: P_n changes its monotonicity, like f, everywhere except small intervals

$$(y_i - \pi/n, y_i + \pi/n)$$

and

$$\|f - P_n\| \leq c(s)\,\omega_3(f, \pi/n),$$

where N is a constant depending only on $\min\limits_{i=1,\ldots,2s}\{y_i - y_{i+1}\},\ c$ and $c(s)$ are constants depending only on $s,\ \omega_3(f,\cdot)$ is the modulus of continuity of the 3-rd order of the function f, and $\|\cdot\|$ is the max-norm.

OLEKSIY KLURMAN, Université de Montréal
Weighted Bernstein and Markov-Nikolskii type inequalities for monotone polynomials of order k

For $n \geq m \geq 0$, we denote

$$M_{q,p}(n,m) := \sup_{P_n \in \mathbb{P}_n} \frac{\|P_n^{(m)}\|_{L_q[-1,1]}}{\|P_n\|_{L_p[-1,1]}},$$

where $\mathbb{P}_n$ denotes the set of all algebraic polynomials of degree $\leq n$. The asymptotic behaviour of $M_{q,p}(n,m)$ was studied by many authors.

In 1926, S. Bernstein showed that

$$\sup_{P_n \in \triangle_n} \frac{\|P_n'\|}{\|P_n\|} = \begin{cases} \frac{(n+1)^2}{4}, & \text{if } n = 2k+1, \\ \frac{n(n+2)}{4}, & \text{if } n = 2k. \end{cases}$$

where the supremum is taken over all monotone polynomials of degree $\leq n$.

Motivated by this result, T. Erdelyi considered analogous problem for $M_{q,p}^{(l)}(n,m)$, where the supremum is taken over all absolutely monotone polynomials of order l and determined the exact asymptotic in the case $p \leq q$.

At the time, A. Kroo and J. Szabados found the exact Markov factors for monotone polynomials of order k in L_1 and L_∞ norms.

In this talk, I will discuss the sharp order of $M_{q,p}^{(l)}(n,m)$ for all values of p, q. I will also discuss the weighted analog of Berstein result for monotone polynomials.

OLEKSANDR MAIZLISH, University of Manitoba

Adaptive Triangulations

Recently developed adaptive methods, where the hierarchy of triangulations is not fixed in advance and depends on the local properties of the function, have received considerable attention in piecewise polynomial approximation over sets of triangulations. We introduce a new adaptive algorithm which, given a characteristic function of some bounded convex domain (a so-called cartoon image), constructs a hierarchical sequence of triangulations that adapt to the local properties of the function. In case of convex domains with piecewise-smooth boundary, this approximation method implies a "theoretically correct" rate of convergence and already outperforms the well-known wavelet methods.

OXANA MOTORNA, Taras Shevchenko National University of Kyiv

On the approximation with Jacobi weights

We will discuss some open problems and results in the the approximation with Jacobi weights

$$q_{\alpha,\beta}(x) := (1-x)^\alpha (1+x)^\beta$$

and their generalizations. Say, for the error

$$E_n^*(f) := \inf_{P_n} \|(f - P_n) q_{-1/2,-1/2}\|_{C[-1,1]}$$

of the best uniform weighted approximation by algebraic polynomials P_n of degree $< n$ of a continuous on $[-1,1]$ function $f \in$ Lip1 (that is $|f(x') - f(x'')| \leq |x' - x''|, \forall x', x'' \in [-1,1]$) the estimate

$$nE_n^*(f) \leq c$$

is well known. However the exact value of the absolute constant c is not known.

Recently the asymptotically exact estimates for the generalized Lebesgue constants of the Fourier-Jacobi sums in the weighted integral norm are obtained.

For a domain $D \subset \mathbf{C}$ with piecewise smooth boundary an analog of the Jacobi weight is

$$q_{\alpha_1,\ldots,\alpha_s}(z) := |z_1 - z|^{\alpha_1} \ldots |z_s - z|^{\alpha_s},$$

where s is the number of the angular points z_j. For such domains and weights complete analogs of the estimates, taking place for the interval, are proved.

Finally, we will discuss the shape preserving approximation with Jacobi weights.

ALEXEI SHADRIN, University of Cambridge, UK
Stable reconstruction from Fourier samples

For an analytic and periodic function f, the m-th partial sums of its Fourier series converge exponentially fast in m, but such rapid convergence is destroyed once periodicity is no longer present (because of the Gibbs phenomenon at the end-points).

We can restore higher-order convergence, e.g., by reprojecting the slowly convergent Fourier series onto a suitable basis of orthogonal algebraic polynomials, however, all exponentially convergent methods appear to suffer from some sort of ill-conditioning, whereas methods that recover f in a stable manner have a much slower approximation rate.

We give to these observations a definite explanation in terms of the following fundamental stability barrier: the best possible convergence rate for a stable reconstruction from the first m Fourier coefficients is root-exponential in m.

IGOR SHEVCHUK, Taras Shevchenko National University of Kyiv
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This is joint work with O. V. Motorna.