OXANA MOTORNA, Taras Shevchenko National University of Kyiv
On the approximation with Jacobi weights

We will discuss some open problems and results in the the approximation with Jacobi weights

$$q_{\alpha,\beta}(x) := (1-x)^{\alpha}(1+x)^{\beta}$$

and their generalizations. Say, for the error

$$E_n^*(f) := \inf_{P_n} \|(f-P_n)q_{-1/2,-1/2}\|_{C[-1,1]}$$

of the best uniform weighted approximation by algebraic polynomials P_n of degree $< n$ of a continuous on $[-1,1]$ function $f \in$ Lip1 (that is $|f(x')-f(x'')| \le |x'-x''|, \forall x', x'' \in [-1,1]$) the estimate

$$nE_n^*(f) \le c$$

is well known. However the exact value of the absolute constant c is not known.

Recently the asymptotically exact estimates for the generalized Lebesgue constants of the Fourier-Jacobi sums in the weighted integral norm are obtained.

For a domain $D \subset \mathbf{C}$ with piecewise smooth boundary an analog of the Jacobi weight is

$$q_{\alpha_1,\ldots,\alpha_s}(z) := |z_1-z|^{\alpha_1}\ldots|z_s-z|^{\alpha_s},$$

where s is the number of the angular points z_j. For such domains and weights complete analogs of the estimates, taking place for the interval, are proved.

Finally, we will discuss the shape preserving approximation with Jacobi weights.