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Percolation, Bootstrap Percolation, and Random Monotone Cellular Automata

Percolation theory, founded by Broadbent and Hammersley in 1957, and r*-neighbour bootstrap percolation*, introduced by Chalupa, Leath and Reich in 1979, have much in common. Thus, on the infinite lattice $\mathbb{Z}^d$ we may start the same way, by selecting vertices of $\mathbb{Z}^d$ with the same probability p to obtain a random set A_0. In percolation a basic question is whether A_0 *percolates*, i.e. the subgraph induced by it has an infinite component; the infimum of those probabilities is the *critical probability* p_c. In r-neighbour bootstrap percolation the set A_0 is only the beginning: considering the vertices in it *infected*, the infection spreads in a very simple way: infected vertices remain infected for ever, and if at any stage a vertex has at least r infected neighbours then itself becomes infected. This system *percolates* if each site is infected eventually. The critical probability p_c is defined as in percolation.

In percolation much work has been done on the critical probability, since it is strictly between 0 and 1 in every non-trivial model; however, in r-neighbour bootstrap percolation the critical probability is of no interest, since every lattice has critical probability 0 or 1.

Recently, Smith, Uzzell and the lecturer introduced a wide-ranging extension of r-neighbour bootstrap percolation: they initiated the study of completely general monotone, local, and homogeneous cellular automata in a random environment. Among other results, they classified these $\mathcal{U}$*-percolation models* into three types, and proved results about the phase transition in two of them. The phase transition in the third type has been clarified by Balister, Przykucki, Smith and the lecturer: in particular, they have shown that in $\mathbb{Z}^2$ these processes have non-trivial critical probabilities. These results have reopened the study of critical probabilities in 'generalized bootstrap processes' on $\mathbb{Z}^2$.

In the lecture I shall survey some classical results in percolation and bootstrap percolation, and will sketch some of the recent results on monotone cellular automata obtained by Balister, Duminil-Copin, Gunderson, Holmgren, Morris, Przykucki, Smith, Uzzell, and myself.