
Algebraic Tools and Design Theory
Outils algébriques et design combinatoire
(Org: Robert Craigen (Manitoba))

ROBERT CRAIGEN, University of Manitoba

Supercharging the 'Unreal' construction for Hadamard and Generalized Hadamard matrices

Over 40 years ago Turyn showed how Hadamard matrices can be derived from "complex Hadamard matrices", which may be described as (Butson-)generalized Hadamard (BH) matrices over the 4th roots of unity. In 2008, de Launey and I introduced a surprising variant in which BH's over the 6th roots of unity give (ordinary) Hadamard matrices, as long as they are "unreal"—that is, they contain no real entries. The question naturally arises as to which kinds of BH lend themselves to a similar construction, and under what constraints. It was easy enough to generate appropriate conditions, but quite another thing to find the necessary raw ingredients to make such a construction work. What we did not know at the time was that only a year earlier a bit of deft work with group representations and field theory provided just what was needed...

SKYPE ROUND TABLE WITH DANE FLANNERY, NUI, Galway

HADI KHARAGHANI, Lethbridge

PETER LOLY, University of Manitoba

"Multimagic" Latin Squares

(research with Adam Rogers and Ian Cameron, Department of Physics and Astronomy)

Boyer has popularized multimagic squares where the rows, columns and main diagonals of full cover magic squares have common "magic" linesums for integer powers p of each element. Bimagic squares corresponding to $p=2$ begin at orders 8 and 9, trimagic for $p=3$ begin at order 12, etc. Rogers observed before 2007 that Latin squares are "multi-semi-magic" for rows and columns. Eggermont called diagonal Latin squares "infinitely-multimagic squares" in a 2004 talk. Now after Nordgren drew our attention to Knut Vik Latin designs (which begin at orders 5,7,11,...) and have the pandiagonal property that all transversals parallel to the main diagonals have a common linesum, we realized that Knut Vik squares are "pandiagonal multimagic" to all integer powers. In fact to any power! Moreover following Heyadat and Federer, and our recent studies (Cameron, Rogers and Loly 2013), these can be compounded to construct Knut Vik designs of multiplicative orders $5*5,5*7,...$ The corresponding matrices are highly singular. Some properties which can be compounded and iterated are discussed. C. Boyer, <http://www.multimagie.com/English/>; Ian Cameron, Adam Rogers and Peter D. Loly, Signatura of magic and Latin integer squares: isentropic clans and indexing", *Discussiones Mathematicae Probability and Statistics*, 33(1-2) (2013) 121-149, or <http://www.discuss.wmie.uz.zgora.pl/ps>; C. Eggermont, <http://www.win.tue.nl/~ceggermo/math/>; A. Heyadat and W. T. Federer *Ann. Statist.*, 3(2)(1975), 445-447, On the Nonexistence of Knut Vik Designs for all Even Orders; R. Nordgren, Pandiagonal and Knut Vik Sudoku Squares, *Mathematics Today*, 49 (2013) 86-87 and Appendix.

FERENC SZOLLOSI, Tohoku University

Modular Combinatorial Designs

In this talk we present a new concept what we call modular symmetric designs and use them to study modular Hadamard matrices. We prove that there exist 5-modular Hadamard matrices of order n if and only if $n \not\equiv 3, 7 \pmod{10}$ and $n \neq 6, 11$. In particular, this solves the 5-modular version of the Hadamard conjecture.

JOHN VAN REES, Dept. of Computer Science, University of Manitoba
The Relationship between (16,6,3)-BIBDs and (25,12) Binary Self-Orthogonal Codes

The $(6\lambda-2, 2\lambda, \lambda)$ -designs are a family where for most of the family their existence is not known. Each incidence matrix for members of this family can be used to generate a binary self-orthogonal code. If there are no such codes "containing" the (v, k, λ) design, then there are no such designs. This was how the $(22, 8, 4)$ -design was shown to be non-existent. Now the next two members of the family do exist but only one non-isomorphic design per parameter is known. Before extensive programming is attempted to find all non-isomorphic designs with these two parameter sets, it would be wise to see the relationship between the previous design in the family; i.e., $(16, 6, 3)$ and the related $(12, 25)$ binary self-orthogonal code. This we do.