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On a Problem of Bourgain Concerning the L_p Norms of Exponential Sums

For $n \geq 1$ let

$$\mathcal{A}_n := \left\{ P : P(z) = \sum_{j=1}^n z^{k_j} : 0 \leq k_1 < k_2 < \cdots < k_n, k_j \in \mathbb{Z} \right\},$$

that is, \mathcal{A}_n is the collection of all sums of n distinct monomials. These polynomials are also called Newman polynomials. Let

$$M_p(Q) := \left(\int_0^1 |Q(e^{i2\pi t})|^p dt \right)^{1/p}, \quad p > 0.$$

We define

$$S_{n,p} := \sup_{Q \in \mathcal{A}_n} \frac{M_p(Q)}{\sqrt{n}} \quad \text{and} \quad S_p := \liminf_{n \rightarrow \infty} S_{n,p} \leq \Sigma_p := \sup_{n \in \mathbb{N}} S_{n,p}.$$

In this talk, we show that

$$\Sigma_p \geq \Gamma(1 + p/2)^{1/p}, \quad p \in (0, 2).$$

The special case $p = 1$ recaptures a recent result of Aistleitner, the best known lower bound for Σ_1 .