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Weighted Bernstein and Markov-Nikolskii type inequalities for monotone polynomials of order k

For $n \geq m \geq 0$, we denote

$$M_{q,p}(n,m) := \sup_{P_n \in \mathbb{P}_n} \frac{\|P_n^{(m)}\|_{L_q[-1,1]}}{\|P_n\|_{L_p[-1,1]}},$$

where $\mathbb{P}_n$ denotes the set of all algebraic polynomials of degree $\leq n$. The asymptotic behaviour of $M_{q,p}(n,m)$ was studied by many authors.

In 1926, S. Bernstein showed that

$$\sup_{P_n \in \triangle_n} \frac{\|P_n'\|}{\|P_n\|} = \begin{cases} \frac{(n+1)^2}{4}, & \text{if } n = 2k+1, \\ \frac{n(n+2)}{4}, & \text{if } n = 2k. \end{cases}$$

where the supremum is taken over all monotone polynomials of degree $\leq n$.

Motivated by this result, T. Erdelyi considered analogous problem for $M_{q,p}^{(l)}(n,m)$, where the supremum is taken over all absolutely monotone polynomials of order l and determined the exact asymptotic in the case $p \leq q$.

At the time, A. Kroo and J. Szabados found the exact Markov factors for monotone polynomials of order k in L_1 and L_∞ norms.

In this talk, I will discuss the sharp order of $M_{q,p}^{(l)}(n,m)$ for all values of p, q. I will also discuss the weighted analog of Berstein result for monotone polynomials.