ALEXEI SHADRIN, University of Cambridge, UK
Stable reconstruction from Fourier samples

For an analytic and periodic function f, the m-th partial sums of its Fourier series converge exponentially fast in m, but such rapid convergence is destroyed once periodicity is no longer present (because of the Gibbs phenomenon at the end-points).

We can restore higher-order convergence, e.g., by reprojecting the slowly convergent Fourier series onto a suitable basis of orthogonal algebraic polynomials, however, all exponentially convergent methods appear to suffer from some sort of ill-conditioning, whereas methods that recover f in a stable manner have a much slower approximation rate.

We give to these observations a definite explanation in terms of the following fundamental stability barrier: the best possible convergence rate for a stable reconstruction from the first m Fourier coefficients is root-exponential in m.