Understanding the plankton dynamics can potentially help us take effective measures to settle the critical issue on how to keep plankton ecosystem balance. In this paper, a nutrient-phytoplankton-zooplankton (NPZ) model is formulated to get insight into understanding the mechanism of plankton dynamics. To account for the harmful effect of the phytoplankton allelopathy, a prototype for a non-monotone response function is used to model zooplankton grazing, and nonlinear phytoplankton mortality is also included in the NPZ model. The main purpose of the paper is to analyze the complex dynamics of the NPZ model, particularly focus on understanding how the phytoplankton allelopathy and nonlinear phytoplankton mortality affect the plankton population dynamics. We first examine the existence of multiple equilibria and provide a detailed classification for the equilibria of the NPZ system, then stability and local bifurcation are also studied. Sufficient conditions for Hopf bifurcation, Bogdanov-Takens bifurcation and zero-Hopf bifurcation are given respectively. Numerical simulations are finally conducted to confirm and extend the analytic results. The theoretical and numerical findings imply that the phytoplankton allelopathy and nonlinear phytoplankton mortality may lead to a rich variety of complex dynamics of the nutrient-plankton system. The results of this study suggest that the effects of the phytoplankton allelopathy and nonlinear phytoplankton mortality should be received additional consideration in understanding the mechanism of plankton dynamics.