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Twisted extensions of Fermat's Last Theorem

Let $x, y, z, p, n, \alpha \in \mathbb{Z}$ with $\alpha \geq 1$, p and $n \geq 5$ primes. In 2011, Michael Bennett, Florian Luca and Jamie Mulholland showed that the equation $x^3 + y^3 = p^\alpha z^n$ has no pairwise coprime nonzero integer solutions provided $p \geq 5$, $n \geq p^{2p}$ and $p \notin S$ where S is the set of primes q for which there exists an elliptic curve of conductor $N_E \in \{18q, 36q, 72q\}$ with at least one nontrivial rational 2-torsion point. I will present a solution that extends the result to include a subset of the primes in S; those $q \in S$ for which all curves with conductor $N_E \in \{18q, 36q, 72q\}$ with nontrivial rational 2-torsion have discriminants not of the form ℓ^2 or $-3m^2$ with $\ell, m \in \mathbb{Z}$. I will further discuss a similar approach used to solve the equation $x^5 + y^5 = p^\alpha z^n$ which in part generalizes work done from Billerey and Dieulefait in 2009. I will also discuss limitations of the method as they extend to further prime exponents.