Progress in Higher Categories
Progrès sur les catégories supérieures
(Org: **Peter LeFanu Lumsdaine** and/et **Michael Shulman** (IAS))

DIMITRI ARA, Radboud Universiteit Nijmegen
On higher quasi-categories

In this talk we will discuss a new model for (∞, n)-categories: the n-quasi-categories. The n-quasi-categories are defined as the fibrant objects of a model structure on the category of presheaves on the category Θ_n of Joyal. For $n = 1$, the notion coincide with the usual quasi-categories. We will compare these n-quasi-categories with the Θ_n-spaces of Rezk. These two models are canonically related in a sense that we will make precise. In particular, we will get two Quillen equivalences between these model structures. For n = 1, we recover the two Quillen equivalences between quasi-categories and complete Segal spaces defined by Joyal and Tierney.

JULIE BERGNER, University of California, Riverside
Group actions on $(\infty, 1)$-categories

Many naturally-arising $(\infty, 1)$-categories are equipped with a group action. A natural question to ask is whether such a structure can be described diagrammatically, especially since many models for $(\infty, 1)$-categories are given by Δ^{op}-diagrams of sets or spaces satisfying some additional assumptions. In joint work with Philip Hackney, we describe diagrams for encoding group actions (or more general category actions) on simplicial structures.

GUILLAUME BRUNERIE, Institute for Advanced Study
An elementary definition of globular weak ∞-groupoids

Combining Grothendieck's definition of weak ∞-groupoids with ideas originating from dependent type theory, one can obtain a simple, explicit and elementary definition of globular weak ∞-groupoids. I will present this definition and discuss some work in progress related to the development of homotopy theory and higher category theory based on globular weak ∞-groupoids. No prior knowledge of type theory will be needed.

TOM FIORE, University of Michigan-Dearborn
Waldhausen Additivity and Approximation in Quasicategorical K-Theory

Waldhausen Additivity, in its general form, says that Waldhausen K-theory sends a split-exact sequence

$$\mathcal{A} \to \mathcal{E} \to \mathcal{B}$$

to a stable equivalence

$$K(\mathcal{E}) \to K(\mathcal{A}) \vee K(\mathcal{B})$$

of spectra. I will sketch a proof for the case that $\mathcal{A}$, $\mathcal{E}$, and $\mathcal{B}$ are Waldhausen quasicategories satisfying mild hypotheses. The method here is to prove the classical theorem in an entirely simplicial way, combining elements of previous proofs, and then carry this proof over to quasicategories. Weak adjunctions between quasicategories and between simplicial categories are also needed. This is joint work with Wolfgang Lück. I will mention the related work by Barwick and Blumberg–Gepner–Tabuada.

Additionally, I will discuss my recent work on a Waldhausen-style Approximation Theorem in quasicategorical K-theory: if an exact functor induces an equivalence of cofibration homotopy categories, then it induces a stable equivalence of K-theory spectra. A corollary is that any exact functor which induces an equivalence of homotopy categories and reflects cofibrations induces a stable equivalence of K-theory spectra. The class of cofibrations, for both Approximation and Additivity, is general.

MORITZ GROTH, Radboud University, Nijmegen
Additive ∞-categories and canonical monoidal structures I

We will describe joint work with David Gepner and Thomas Nikolaus, in which we study additive ∞-categories. Examples of such ∞-categories are given by commutative group objects, and these examples are generic in a certain sense. In the framework of presentable ∞-categories, additive ∞-categories define a localization, and there are similar such results for pointed, preadditive, and stable ∞-categories. This localization picture will be analyzed further in the talk by Thomas Nikolaus, in which also some applications will be mentioned.

NICK GURSKI, University of Sheffield
Semi-strictness via factorization

Gray-categories are the most studied semi-strict form of higher category, and the tensor product that defines them can be obtained from a factorization system on $\mathbf{2Cat}$. Finding an analogous tensor product on $\mathbf{GrayCat}$ turns out to be a very hard problem, but one can define a slightly weaker notion of semi-strict n-category inductively using the standard machinery of locally presentable categories together with Garner's approach to weak morphisms via cofibrant replacement. I will describe such an inductive definition, which is joint work with John Bourke.

CLAUDIO HERMIDA,
Representable n-Multicategories

We pursue the treatment of higher-dimensional weak categories via universal properties exploring representability for n-multicategories. We follow the theory set up in [2], [1] where we recaptured monoidal categories as representable 1-multicategories. At dimension 2, as expected, we recapture monoidal bicategories (with respect to Gray-tensor product).

[1] Claudio Hermida. From coherent structures to universal properties. Journal of Pure and Applied Algebra, 165(1):7–61, 2001. preprint available math.CT/0006161.

[2] Claudio Hermida. Representable multicategories. Adv. Math., 151(2):164–225, 2000.

[3]Tom Leinster. Higher operads, higher categories, volume 298 of London Mathematical Society Lecture Note Series. Cambridge University Press, Cambridge, 2004.

ANDRÉ JOYAL, UQAM
Categorical Aspects of Homotopy Type Theory

Homotopy type theory may eventualy provide a formal language for higher category theory. Our goal is to give a categorical analysis/presentation of type theory. We show that it can be formalised with a single weak factorisation system (the Gambino-Garner system).

JOACHIM KOCK, Universitat Autònoma de Barcelona
Incidence algebras and Möbius inversion in Rezk categories and decomposition spaces

I'll explain how the classical theory of incidence algebras of locally finite posets (Rota) and Möbius categories (Leroux) can be generalised to higher categories, leading to the new notion of decomposition space: it is a simplicial (infinity) groupoid satisfying an exactness condition weaker than the Segal condition, expressed in terms of generic and free maps in Delta. Just as the Segal condition expresses up-to-homotopy composition, the new condition expresses decomposition (and as everybody knows from watch repairs, it is much easier to decompose than to compose). New examples covered by the theory include the Faà di Bruno and Connes-Kreimer bialgebras, the Lawvere-Menni category of Möbius intervals which contains the universal Möbius function (but is not itself a Möbius category), and Hall algebras: the Waldhausen S-construction of abelian (or stable infinity) categories are decomposition spaces, and their incidence algebras are Hall algebras. This is joint work with Imma Gálvez and Andy Tonks.

THOMAS NIKOLAUS, Universitaet Regensburg
Additive ∞-categories and canonical monoidal structures II

We continue the discussion started in the talk of Moritz Groth. Given a presentable, symmetric monoidal ∞-category C, the ∞-category of E_∞-groups in C admits a canonical and unique tensor product. This generalizes the tensor product of abelian groups, and there are similar results for monoids and other contexts. We conclude by indicating applications to multiplicative infinite loop space theory and algebraic K-theory (any maybe equivariant homotopy theory if time permits).

Our main tools are stability results for algebraic structures under basechange and the theory of smashing localizations applied to the ∞-category of presentable ∞-categories.

BOB PARÉ, Dalhousie University
The "Triple Category" of Bicategories

It is well known that bicategories, lax functors and lax transformations don't form a bicategory or even a tricategory if we include modifications. Yet these are important concepts. We generalize the notion of lax transformation, and study the resulting structure. There is a lax version of interchange which we consider to be the central feature.

DORETTE PRONK, Dalhousie University
Weakly globular double categories

In this talk I will introduce weakly globular double categories. They form a version of weak 2-categories in which the globularity condition, rather than the unit or associativity axioms, has been relaxed: instead of having a set of objects these weak 2-categories have a category of objects which is weakly equivalent to a discrete category. This is formalized by considering the category of objects as the vertical arrow category in a (strict) double category. There are two ways of adding arrows and 2-cells to construct a 2-category of weakly globular double categories, one of which gives a 2-category which is biequivalent to the 2-category of bicategories with pseudo-morphisms and icons.

If there is enough time I will show some illustrate some of the features of this notion of weak 2-category by giving the construction of the weakly globular double category of fractions for a category with a chosen class of arrows. This weakly globular double category of fractions has a universal property with respect to both 2-category structures for weakly globular double categories. Furthermore, it has the property that it is locally small, in the sense that it has horizontal and vertical hom-sets and sets of double cells with given domains and codomains. This is important when considering its classifying space for instance.

This is joint work with Simona Paoli.

EMILY RIEHL, Harvard University
The formal category theory of quasi-categories

Quasi-categories (aka ∞-categories) are convenient models of categories weakly enriched in spaces. Analogs of the standard categorical theorems involving limits and colimits, adjunctions, equivalences, and so forth have been proven by Joyal, Lurie, and others. The goal of this talk is to describe a new ground-level approach that allows for "formal" re-proofs of these facts that requires only very mild model categorical input and hence can be interpreted in other higher categorical contexts. The general strategy involves mediating between simplicially enriched universal properties and 2-categorical universal properties. A key technical observation is that there exist certain weak 2-limits in the strict 2-category of quasi-categories whose universal properties is robust enough to do formal category theory. We illustrate with as many sample proofs as time permits. This is a progress report on ongoing joint work with Dominic Verity.

DANIEL SCHÄPPI, The University of Chicago
Two-dimensional Morita theory and Galois cohomology

To each commutative ring we can naturally associate three groups: the group of units, the Picard group (the group of isomorphism classes of invertible modules), and the Brauer group (consisting of Morita equivalence classes of Azumaya algebras). Starting with an arbitrary commutative ring, John Duskin constructed a topological space whose homotopy groups are isomorphic to these three groups. Ross Street gave a construction of this space as the nerve of a monoidal bicategory.

If we work with a field, the unit, Picard, and Brauer group coincide with the first three Galois cohomology groups. In my talk I will outline the construction of a monoidal tricategory which, conjecturally, has the property that the homotopy groups of its nerve coincide with the first four Galois cohomology groups. The key ingredient is a generalization of Morita theory and Azumaya algebras to the context of two-dimensional category theory. This is work in progress, joint with Evan Jenkins.

URS SCHREIBER, University Nijmegen
Synthetic quantum theory in higher cohesive toposes

There are two traditional mathematical formalizations of quantum physics via quantization: "algebraic deformation quantization" and "geometric quantization". I discuss how the latter has a natural axiomatic ("synthetic") formulation in those higher toposes which are equipped with two adjoint triples of higher idempotent modalities – a joint refinement of what Lawvere had called "synthetic differential geometry" and "cohesion". This yields synthetic re-derivations of classical and of more recent results in geometric quantization, and lifts them from quantum mechanics to local ("extended") quantum field theory in higher dimension. I close by indicating examples of Chern-Simons type field theories. The example in dimension 2 recovers C-star algebraic deformation quantization as its "holographic dual"; the example in dimension 3 holographically recovers twisted equivariant K-theory. The example in dimension 7 relates to current questions in non-perturbative string theory.

(Related material is available at ncatlab.org/schreiber/show/Higher+geometric+prequantum+theory)

MARK WEBER, Macquarie University
Weak n-categories with strict units via iterated enrichment

This is joint work with Michael Batanin and Denis-Charles Cisinski.

The notion of a reduced higher operad is introduced to formalise the idea that the higher categorical structures being considered have a unique operation of each "unit type". Garner's machinery of algebraic weak factorisation systems then gives rise to a notion of contractibility for reduced operads, which includes the feature that the unique unit operations act as units with respect to the other operations. There is an initial reduced n-operad that is contractible in this sense, thus giving an operadic definition of weak n-category with strict units. Moreover, there is a lax tensor product on the category of algebras of this operad, and enriched categories with respect to this lax tensor product can be identified with weak (n+1)-categories with strict units.