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Incidence algebras and Möbius inversion in Rezk categories and decomposition spaces

I'll explain how the classical theory of incidence algebras of locally finite posets (Rota) and Möbius categories (Leroux) can be generalised to higher categories, leading to the new notion of decomposition space: it is a simplicial (infinity) groupoid satisfying an exactness condition weaker than the Segal condition, expressed in terms of generic and free maps in Delta. Just as the Segal condition expresses up-to-homotopy composition, the new condition expresses decomposition (and as everybody knows from watch repairs, it is much easier to decompose than to compose). New examples covered by the theory include the Faà di Bruno and Connes-Kreimer bialgebras, the Lawvere-Menni category of Möbius intervals which contains the universal Möbius function (but is not itself a Möbius category), and Hall algebras: the Waldhausen S-construction of abelian (or stable infinity) categories are decomposition spaces, and their incidence algebras are Hall algebras. This is joint work with Imma Gálvez and Andy Tonks.