Hopf Algebras and Tensor Categories
Algèbres de Hopf et catégories tensorielles
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YURI BAHTURIN, Memorial University of Newfoundland
Group Gradings on Nilpotent Lie Algebras

Some problems in Differential Geometry makes it interesting to classify gradings on nilpotent Lie algebras. In this talk I present joint results with Michel Goze and Elizabeth Remm about the classification of gradings by abelian groups on filiform Lie algebras of nonzero rank.

JUAN CUADRA, University of Almeria
New examples of Hopf algebras with nonzero integral

The Haar measure on a compact group induces a linear functional $\int$ on the Hopf algebra of its representative functions. The invariance property of the Haar measure reads as a condition on $\int$ that can be expressed in Hopf algebraic terms. Sweedler defined an algebraic notion of integral for Hopf algebras using this condition. Hopf algebras having a nonzero integral are also called co-Frobenius.

The quantized coordinate algebra $\mathcal{O}_q(G)$ of a simple algebraic group G is one of the most relevant examples of co-Frobenius Hopf algebras. When q is a root of unity, the usual coordinate algebra $\mathcal{O}(G)$ is a central Hopf subalgebra of $\mathcal{O}_q(G)$ and $\mathcal{O}_q(G)$ is finitely generated (and free) as a module over $\mathcal{O}(G)$. Moreover, $\mathcal{O}(G)$ coincides with the Hopf socle of $\mathcal{O}_q(G)$. Based on this example, Andruskiewitsch and Dăscălescu asked in [*Co-Frobenius Hopf algebras and the coradical filtration*. Math. Z. **243** (2003), 145-154] whether any co-Frobenius Hopf algebra is finitely generated as a module over its Hopf socle.

In this talk we will introduce a new family of co-Frobenius Hopf algebras that answers in the negative this question. The results that will be presented are part of a joint work with N. Andruskiewitsch (National University of Córdoba, Argentina) and P. Etingof (Massachusetts Institute of Technology, USA). Arxiv:1206.5934.

GASTON GARCIA, Universidad Nacional de La Plata
Pointed Hopf algebras over finite simple groups of Lie type

The general question we are dealing with is the classification of finite-dimensional complex pointed Hopf algebras H over a finite simple group. We say that a finite group G *collapses* when every finite-dimensional pointed Hopf algebra H, with $G(H) \simeq G$ is isomorphic to $\Bbbk G$.

This problem was attacked by several authors obtaining, among others, the following results: If $G \simeq \mathbb{Z}/p$ is simple abelian, then the classification is known. In case G is non-abelian, the problem is far from being solved. If $G \simeq \mathbb{A}_m$, $m \geq 5$ is alternating, then G collapses. If G is a sporadic simple group, then G collapses, except for the groups $G = Fi_{22}$, B, M.

A remarkable fact is that the class of braided vector spaces corresponding to pointed Hopf algebras with non-abelian group can be identified as those constructed from racks and cocycles. In this sense, we say that a rack X collapses if the Nichols algebra associated to it is infinite-dimensional for all cocycles. Given a simple non-abelian group G, as a first step one has to determine all conjugacy classes in G that collapse.

If G is a finite simple group of Lie type, one may reduce the study to unipotent or semisimple conjugacy classes. In this talk, we present our study on unipotent conjugacy classes in Steinberg groups, based on the explicit presentation of our result on $\mathbf{PSL}_n(q)$.

This talk is based on a joint work with N. Andruskiewitsch and G. Carnovale.

ALLEN HERMAN, University of Regina
Brauer-Clifford groups for Hopf Algebra actions and coactions

If G is a group, and S is a commutative G-algebra, the Brauer-Clifford group $BrClif(S,G)$ is a group of equivalence classes of S-Azumaya G-algebras with a fixed nontrivial action of G on S. These were introduced by Alexandre Turull the context of his study of questions concerning various extensions of the McKay conjecture in the representation theory of finite groups. Work of Dipra Mitra and the speaker showed that this Brauer-Clifford group is an equivariant Brauer group for the symmetric monoidal category of S*G-modules for the skew group ring, where S is any commutative G-algebra. This perspective allows one to define a Brauer-Clifford group in the category of $S\#H$-modules for the smash product when H is a cocommutative Hopf algebra, and a dual Brauer-Clifford group involving an H-coaction when H is commutative. I will introduce these new Brauer-Clifford groups and describe their basic properties. This is joint work with Thomas Guedenon.

MIODRAG IOVANOV, University of Iowa and University of Bucharest
On co-Frobenius Hopf algebras and coalgebras

A conjecture of Andruskiewitsch and Dascalescu in 2004 stated that a Hopf algebra with non-zero integrals has finite coradical filtration. This was first observed by Radford in the '70's for pointed co-Frobenius Hopf algebras. Recently, a short and elegant proof for (a stronger version) of this conjecture was given by Andruskiewitsch, Cuadra and Etingof, as well as an extension to Frobenius tensor categories of subexponential growth. We extend their ideas to give a proof for a version of this statement in the setting of co-Frobenius coalgebras; in particular, some new examples of tensor categories where the Andruskiewitsch-Cuadra-Etingof result works are obtained. We also examine counter-examples for other versions of this conjecture for general co-Frobenius coalgebras.

YEVGENIA KASHINA, DePaul University
Semisimple Hopf Algebras of dimension 32

In this talk we will discuss classification of semisimple Hopf algebras of dimension 32. We will fix an abelian group of grouplike elements of order 16 and describe all nonisomorphic semisimple Hopf algebras of dimension 32 with this group of grouplike elements.

MIKHAIL KOTCHETOV, Memorial University of Newfoundland
Graded modules over simple Lie algebras with a group grading

The Cartan decomposition of a semisimple Lie algebra with respect to a Cartan subalgebra can be regarded as a grading by a free abelian group. Gradings on Lie algebras by various abelian groups arise in the theory of symmetric spaces, Kac-Moody algebras, and color Lie superalgebras. In the 1960s, V. Kac classified all gradings by cyclic groups on finite-dimensional simple Lie algebras over complex numbers. Recently, with efforts of several authors, the classification of gradings by an arbitrary abelian group G has been obtained for any classical simple Lie algebra L, except of type D_4, over an algebraically closed field of characteristic different from 2. Given such a grading on L, it is natural to study graded L-modules. In characteristic 0, any finite-dimensional graded L-module is a direct sum of simple graded L-modules. We will describe finite-dimensional simple graded L-modules and consider the following related problem: which of the finite-dimensional L-modules admit a G-grading making them graded modules? This is a joint work with Alberto Elduque.

LEN KROP,
Isomorphism classes in a class of Abelian extensions

We give a general method for classification of isomorphism classes of Hopf algebra extensions of the group algebra kC_p of a cyclic group kC_p (p prime) by the group algebra kG of a finite abelian p-group G. Our principal aims are: (i) A structure theorem for the second Hopf cohomology group, (ii) An isomorphism criterion for two extensions, (iii) A bijective correspondence between isoclasses of extensions and orbits of $Aut_{C_p}(G) \times AutC_p$ in the second Hopf cohomology group, (iv) The number of orbits in case of commutative extensions or $G = C_p \times C_p$.

AARON LAUVE, Loyola University Chicago

Convolution Powers of the Identity in Graded Connected Hopf Algebras

We answer the question, "what are the eigenvalues of the k-th convolution power of the identity operator acting on a graded connected Hopf algebra?" Well, the answer is simple to state, "powers of k (for any integer k)." Thus, we'll spend most of the time talking about eigenvalue multiplicity, combinatorial implications, and a connection to the classical Frobenius–Schur indicators. This is joint work with Marcelo Aguiar (Texas A&M).

SARA MADARIAGA, University of Saskatchewan

Special identities for the pre-Jordan product in the free dendriform algebra

The Hopf algebra of planar binary trees studied by Loday and Ronco in 1998 can be regarded as the free dual dialgebra introduced by Loday in 1998, now known as the free dendriform algebra. This algebra has two operations whose sum is associative and can be obtained from a Rota-Baxter algebra as shown independently by Aguiar and Ebrahimi-Fard.

Pre-Lie and pre-Jordan algebras were introduced in the study of the Yang-Baxter equation and can be obtained from a Rota-Baxter operator on Lie and Jordan algebras respectively. Dendriform algebras with the pre-Lie or the pre-Jordan product are also pre-Lie or pre-Jordan algebras respectively.

The problem of constructing universal dendriform envelopes of pre-Lie algebras with an injective natural map is still open. We approached the same problem for pre-Jordan algebras from the point of view of polynomial identities. We found special identities for the pre-Jordan product in the free dendriform algebra, which means that universal dendriform envelopes for pre-Jordan algebras do not have in general an injective natural map.

This is a joint work with Prof. Murray Bremner.

AKIRA MASUOKA, University of Tsukuba

Hopf algebraic techniques applied to affine algebraic supergroups

I will report my recent results on affine algebraic supergroups, joint with Alexandr Zubkov, Taiki Shibata and Craig Pastro, emphasizing the Hopf algebraic techniques applied. The results will include a basic result on the quotient sheaf G/H (joint with Zubkov), a category equivalence between the Harish-Chandra pairs and the affine algebraic supergroups (by myself), a Hopf algebraic construction of the Chevalley supergroups over $\mathbb{Z}$ (joint with Shibata), and some results on integrals (joint with Patro).

SUSAN MONTGOMERY, University of Southern California

Brauer characters for representations of bismash products

For a finite group G, Brauer characters give a way of studying irreducible representations in characteristic $p > 0$, by "lifting" information to characteristic 0. In joint work with A. Jedwab, we extend the notion of Brauer characters and its properties to the case of a bismash product Hopf algebra $H_k = k^G \# kF$ of groups F, G over a field k; such an algebra is constructed from a factorizable group $L = FG$, where F, G are subgroups of L and $F \cap G = 1$. Since the representations of H come from representations of certain stabilizer subgroups F_x of F, our method is to lift the classical Brauer characters from these F_x to a character of H. We define a Cartan matrix analogously as for groups and show its determinant is a power of p. We prove the analog of a theorem of Thompson (1986) on Frobenius-Schur indicators:

THEOREM: Let k be an algebraically closed field of odd characteristic and let C be the complex numbers. Then if H_C is "totally orthogonal" (that is every irreducible representation has a symmetric, non-degenerate H_C-invariant bilinear form), the same is true for H_k.

We apply the theorem and our previous work with Jedwab over C to show that if k is as in the theorem and $H_k = k^{C_n} \# kS_{n-1}$ is the bismash product constructed from the standard factorization of the symmetric group $S_n = S_{n-1}C_n$, then H_k is totally orthogonal.

SIU-HUNG NG, Iowa State and Cornell University
Cauchy's Theorem for Spherical Fusion Categories

Cauchy's theorem for finite groups asserts that the exponent and the order of a finite group have the same prime factors. The theorem has been generalized to semisimple Hopf algebras as well as quasi-Hopf algebras, in which the dimensions of these algebras replace the role of the orders of finite groups. In this talk, we will discuss a generalization of Cauchy's theorem for spherical fusion categories and its implication to finiteness conjecture of modular categories. This is joint work with P. Bruillard, E. Rowell and Z. Wang.

PETER SCHAUENBURG, Université de Bourgogne, Dijon
Weak Hopf Algebras Associated to Tambara-Yamagami Categories

Tambara and Yamagami have introduced a now well-known family of fusion categories whose fusion rules, involving a single non-invertible simple object, are given in terms of a nondegenerate bicharacter on the, necessarily abelian, group of invertible objects. In some cases, essentially determined by Tambara, these categories are equivalent to module categories of Hopf algebras. In other cases there are obstacles, the most obvious being noninteger dimension of the noninvertible simple object. However, even if this dimension is an integer, quasi-Hopf algebras instead of ordinary Hopf algebras may be needed. A general result of Hayashi guarantees that every fusion category is equivalent to the representation category of a weak Hopf algebra. However, the weak Hopf algebras provided by Hayashi's construction are rather large. Not only is their dimension rather large compared to the dimension of the category. Also the size of the source and target counital subalgebras (which could be seen as measuring the difference between an ordinary and a weak Hopf algebra) is somewhat generous: These semisimple algebras have the same number of simple factors as there are simple objects in the category. We investigate some cases in which Tambara-Yamagami categories cannot be described by ordinary Hopf algebras, but smaller weak Hopf algebras than those given by Hayashi's general procedure can be found; these will be smaller both in dimension (though still larger than the dimension of the category) and in the number of components of the base algebra (though this will still be greater than one).

YORCK SOMMERHAUSER, University of South Alabama
Semilinear Actions of General Linear Groups on Character Rings of Hopf Algebras

It has been shown by Y. Zhu and the speaker that the action of the modular group $\mathrm{SL}(2,\mathbb{Z})$ on the character ring of a semisimple factorizable Hopf algebra factors over the reduced modular group $\mathrm{SL}(2,\mathbb{Z}_N)$ of 2×2-matrices with entries in the finite ring $\mathbb{Z}_N$ of integers modulo N, where N is the exponent of the Hopf algebra, under the assumption that the base field has characteristic zero and that the value of an integral on the inverse Drinfel'd element differs from its value on the Drinfel'd element itself by at most a sign.

Here, the reduced modular group acts via linear maps. However, as we explain in the talk, this action can be extended to an action of the general linear group $\mathrm{GL}(2,\mathbb{Z}_N)$ if one does not only consider linear maps, but also semilinear maps, where 'semilinear' means that the scalars are modified by the action of the Galois group $\mathrm{Gal}(\mathbb{Q}_N/\mathbb{Q})$ of the cyclotomic field. This action of the general linear group also provides a better understanding of a certain Galois condition satisfied by the Drinfel'd element. The talk is based on a recent article (Adv. Math. 236 (2013), 158-223) written jointly with Y. Zhu. We present the results using modular data.

HAMID USEFI, Memorial University
The interplay between group algebras and enveloping algebras

Let $\mathbb{F}G$ and $\omega(G)$ be the group algebra and augmentation ideal of a group G over a field $\mathbb{F}$ of characteristic p, respectively. We first recall a theorem of Quillen stating that the graded algebra associated to $\mathbb{F}G$ is isomorphic as an algebra to the enveloping algebra of the restricted Lie algebra associated to the dimension series of G. We then extend a theorem of Jennings that provides a basis for the quotient $\omega^n(G)/\omega^{n+1}(G)$ in terms of a basis of the restricted Lie algebra. Finally, we present some applications of these theorems.

YINHUO ZHANG, University of Hasselt
Green rings of rank one pointed Hopf algebras of nilpotent type

Let H be a finite dimensional pointed rank one Hopf algebra of nilpotent type, and $G = G(H)$, the group of group-like elements of H. We study the finite dimensional indecomposable H-modules and establish the Clebsch-Gordan formulas for the decompositions of the tensor products of indecomposable H-modules. It turns out that the Green ring $r(H)$ is commutative and generated by one variable over the Grothendieck ring of the group algebra kG modulo one relation. The Jacobson radical of $r(H)$ is completely determined and is a principal ideal of $r(H)$ generated by one element. As an example, we shall describe the Green ring of the pointed rank one Hopf algebra H with $G(H)$ a Dihedral group.