Ergodic Theory, Dynamical Systems and Applications
Théorie ergodique, systèmes dynamiques et applications
(Org: **Pawel Gora** (Concordia) and/et **Shafiqul Islam** (UPEI))

ERIK M. BOLLT, Clarkson University
Finite Time Coherent Sets with Nonhyperbolic Boundaries

Will present a new perspective on finite time coherent sets of nonautonomous. We also present a method based on a continuation principle of finite time nonhyperbolic points to develop curve segments which may be used to define boundaries of coherent sets. Theory and examples will be presented.

CHRISTOPHER J. BOSE, University of Victoria
Asymptotics for random intermittent maps

We study a class of random transformations built over finitely many intermittent maps sharing a common indifferent fixed point. A more-or-less standard treatment via Markov extensions can be used to show how the map with the fastest relaxation rate dominates the asymptotics, in particular, the rate of correlation decay. In this talk I will report on recent joint work in this direction with Wael Bahsoun and Yuejiao Duan, University of Loughborough.

ELENA BRAVERMAN, University of Calgary
On stochastic perturbations of chaotic dynamical systems: is stabilization possible?

Simple one-dimensinal maps can experience a period doubling route to chaos. However, under a stochastic perturbation with a positive mean, this process can be reversed in the sense that a map has a stable blurred 2-cycle for large enough values of the parameter. In the lecture the limit dynamics of this cycle will be described. It will be demonstrated that most well-known population dynamics models (Ricker, truncated logistic, Hassel and May, Bellows maps) have this stable blurred 2-cycle. For a general type of maps, in addition, there may be a blurred stable area near the equilibrium. This is a joint work with A. Rodkina (University of the West Indies, Jamaica).

CARL DETTMANN, University of Bristol, UK
Escape and diffusion through small holes

A dynamical system may be "opened" by allowing trajectories to leak out through one or more holes (subsets of phase space). Given a distribution of initial conditions, we study the probability of remaining within the system as a function of time and the size and position of the hole(s). A chain of systems linked by their holes can also model deterministic diffusion. Recent results for escape and diffusion in one-dimensional expanding maps will be discussed, including the first expansion for the escape rate beyond linear order in hole size, an exact additivity formula for diffusion coefficients and new relations between escape, diffusion and periodic orbits. Connections will be made with particles escaping from containers with small holes: Open billiards.

MARLENE FRIGON, Université de Montréal
Boundary value problems for systems of second order equations on times scales.

We present existence results to boundary value problems for systems of second order equations on times scales. We first consider the case where the right member of the equation does not depend on x^{Δ}. Then we present a result for the more general case where the right member of the equation is $f(t, x(\sigma(t)), x^{\Delta}(t))$. Our results rely on a new notion of solution-tube of systems of second order equations on times scales.

PAWEL GORA, Concordia University
Consecutive maxima maps

τ is a continuous map on a metric compact space X. For a continuous function $\phi : X \to \mathbb{R}$ we consider a 1-dimensional map T (possibly multi-valued) which sends a local ϕ-maximum on τ trajectory to the next one: consecutive maxima map. The idea originated with famous Lorenz's paper. We prove that if T has a horseshoe disjoint from fixed points, then τ is in some sense chaotic, i.e., it has a turbulent trajectory and thus a continuous invariant measure.

MUDASSAR IMRAN,
Optimal Dosing Strategies against Susceptible and Resistant Bacteria

Abstract: Antibiotic modelling is concerned with the problem of finding efficient and successful dosing techniques against bacterial infections. In this study, we model the problem of treating a bacterial infection where the bacteria is divided into two sub-populations: susceptible and resistant. The susceptible type may acquire the resistance gene via the process of conjugation with a resistant bacterium cell. After proposing a model for horizontal gene transfer, we find the steady state solutions under an antibiotic protocol of discrete periodic doses and analyse their stability. We also prove the result that guarantees the persistence of bacteria. In addition, efficient treatment strategies are devised that ensure bacteria elimination while minimizing the quantity of antibiotic used. Such treatments are necessary to decrease the chances of further development of resistance in bacteria and to minimize the overall treatment cost. We consider the cases of varying antibiotic costs, different initial bacterial densities and bacterial attachment to solid surfaces, and obtain the optimal strategies for all the cases. The results show that the optimal treatments ensure disinfection for a wide range of scenarios.

BRIAN INGALLS, University of Waterloo
Sensitivity Trade-offs in Systems Biology

The stabilizing effect of negative feedback is key to biological self-regulation (homeostasis). Feedback allows a system to maintain its preferred behaviour in an unpredictable environment. The prevailing wisdom is that negative feedback typically stabilizes a system (making it less sensitive to external perturbations), while positive feedback is destabilizing (i.e. it increases sensitivity). Of course, negative feedback can also generate instability, for example in producing oscillations. However, even when acting to improve a system's robustness, negative feedback typically redistributes sensitivity within a network, rather than directly reducing it. In some cases, this redistribution is governed by an explicit constraint: a conservation of sensitivity. This talk will introduce sensitivity conservation statements commonly used in control engineering and molecular systems biology, and introduce a unifying formulation.

TOMOKI INOUE, Ehime University
Invariant measures for random maps with continuous random parameters

We consider a family of transformations with a random parameter and study a random dynamical system in which one transformation is randomly selected from the family and applied on each iteration. The parameter space may be of cardinality continuum. The selection of the transformation need not be independent of the position in the state space. We study the existence of absolutely continuous invariant measures for random maps on an interval under some conditions. We also study some estimates of the stationary densities for some examples.

KAMRAN KAVEH, University of Waterloo
Population dynamics of cancer stem cell

Evolutionary modelling of biological systems - in a Darwinian sense - as a discrete or continuous time dynamical systems has been an important part of mathematical biology particularly in genetics and cancer biology. In cancer biology. an analysis of the dynamics of an invasive mutant stem cell introduced in a tissue compartment through a gain/loss-of-function mutation

can be critical to determine the initiation and progression of cancer. We construct a stochastic dynamical system of two populations of stem cells (normal and mutant) and their progenitors (differentiated cells). By finding the fixed points of such a stochastic dynamical system and analyzing the stability of its fixed points, we find the conditions required for a mutant stem cell to take over the whole tissue or conversely to become extinct. An important observation of this model is the fact that for a large set of parameter values there is a chance of the two stem cell phenotypes co-existing. This is important as population co-existence is normally a feature of evolutionary game theoretical models. We discuss the extension of the above model in the presence of spatial structure and the resultant effect on the population dynamics. We also briefly discuss the similarities of stem cell dynamics to evolutionary game theoretical models.

ZHENYANG LI, Concordia University
W-shaped maps and slope condition for stability of acim

We will talk about $W-$shaped maps. It was conjectured that instability of the absolutely continuous invariant measure (acim) can result only from the existence of small invariant neighbourhoods of the fixed critical point of the limit map. We will show that the conjecture is not true by constructing a family of $W-$shaped maps, for which the standard bounded variation method cannot be applied. We then generalize this result. Motivated by the above results, we introduce the harmonic average of slopes condition, which is helpful to deal with the stability for acims of family of piecewise expanding maps.

JUNLING MA, University of Victoria
Vaccination on random contact networks

Contact network more realistically represent who contacts who in the population than commonly used homogeneous mixing epidemiological models. We show that the level of herd immunity required on a random contact network is the same as in a homogeneously mixed population if individuals are uniformly chosen for vaccination. However, it is much more efficient to vaccinate individuals with a probability proportional to the number of contacts that one has. The results are derived from a compartmental modeling framework for node removal on contact networks, but the analysis is closely tied to probability generating functions that can be studied with a PDE, and the results have a intuitive probability theory explanation.

TUFAIL MALIK, Khalifa University of Science, Technology and Research
The Impact of a Vaccine and Pap Screening on the Human Papillomavirus Transmission and Cervical Cancer

The talk will address the problem of the transmission dynamics of human papillomavirus in a population. A new sex-structured model, which takes into account the associated multiple cervical intraepithelial neoplasia stages, will be used to assess the combined impact of Pap cytology screening and a vaccine on the disease dynamics and the associated dysplasia. Rigorous qualitative analysis will be presented. Simulation results, using a realistic set of parameter values, will also be discussed.

MAKOTO MORI, Nihon University
On the Spectrum of Perron-Frobenious Operators on Continuous Functions

We consider piecewise linear and Markov transformations with same slope on an interval. As is well-known, when we restrict the Perron–Frobenius operator associated with the transformation to the set of functions with bounded variation, the spectral radius equals 1 and the essential spectral radius equals the reciprocal of the slope. On the other hand, if the transformation is Markov, the dynamical zeta function is a rational function. Thus, when we consider a suitable domain, there exists possibility that the essential spectral radius may become smaller. In this article, we will study the cases when we restrict its domain to the set of piecewise continuous functions.

WAYNE NAGATA, University of British Columbia
Wave train selection behind predator invasions in a population model with nonlocal prey competition

Periodic wave trains can form behind invasion fronts in predator-prey reaction-diffusion models. Typically there exists an infinite family of such wave trains, arising through Hopf bifurcations and characterized by their wavelengths, but only one selected wave train is observed behind an invasion front in numerical simulations. We develop a criterion that predicts the wavelength of the selected wave train behind invasions, and apply it to a population model with a nonlocal term. Comparing our predictions with the results of numerical simulations, we find the wavelengths predicted are accurate for a range of parameter values. We also study the spectral stability of the wave trains.

ISRAEL NCUBE, Memorial University
Stability switching and Hopf bifurcation in a multiple-delayed neural network

We consider a network of three identical neurons incorporating distributed and discrete signal transmission delays. The model for such a network is a system of coupled nonlinear delay differential equations. Two cases of a single Hopf bifurcation may occur at the trivial equilibrium of the system, a result of the symmetry of the network. These are the simple and the double root single Hopf bifurcations. The presentation examines the simple root case, arriving at a number of stability results.

TOMASZ SZAREK, Institute of Mathematics, University of Gdansk
Stability of Random Dynamical Systems

Our talk is devoted to the problem of stability of random dynamical systems. We are aimed at presenting criteria for the existence of an invariant measure for Markov processes corresponding to function systems and stochastic differential equations. The issue of stability and the rate of convergence will be addressed also. We shall show some consequences following from the exponential rate of convergence for Markov processes under considerations, i.e., the Central Limit Theorem $(\mathbf{CLT})$ and the Law of the Iterated Logarithm $(\mathbf{LIL})$.

GEOFF WILD, The University of Western Ontario
Evolutionary Dynamics and Helping a Neighbour

Animals often incur a cost to provide help to their neighbours, and at first glance this observation strikes many people as strange. In a "dog-eat-dog" world, why on earth would any individual be willing to decrease its own fitness to increase that of a competitor? It turns out there are good reasons why helpful behaviours might be favoured in nature, and in this talk I will explore one such reason: kinship. I will briefly outline a branching-processes model for the emergence of helping. Among other things, I will demonstrate how details of the long-term behaviour of the branching process can be understood in terms of biologically meaningful quantities like relatedness and reproductive value. I will also outline a simple ODE model for the emergence of cooperative breeding systems. Such systems occur whenever individuals help raise offspring produced by their neighbours, and they represent one of the most conspicuous instances of helping in nature.