Analysis, Geometry and Topology on Fractals, Wavelets and Self-Similar Tilings
Analyse, géométrie et topologie sur les fractales, ondelettes et pavages autosimilaires
(Org: **Eva Curry** (Acadia), **Franklin Mendivil** (Acadia) and/et **Tara Taylor** (StFX))

JOE CHEN, Cornell University
Recent progress on Gaussian free fields on fractals

A Gaussian free field on a discrete (resp. continuous) space X is a centered Gaussian process whose covariance is given by the Green's function for simple random walk (resp. Brownian motion) on X. As such the properties of this random field are intimately connected to the Laplacian on X. While the free fields on $\mathbb{R}^d$ or $\mathbb{Z}^d$ have been well studied for decades, rigorous investigations of free fields on fractals have just begun to take off, taking advantage of the potential theoretic techniques developed for studying analysis on fractals.

In this talk I will describe several recent results concerning the geometry of the free field on fractals or fractal-like graphs. These include:

(i) Maxima of the free field on recurrent fractal graphs. (Kumagai-Zeitouni)

(ii) Entropic repulsion of the free field on high-dimensional Sierpinski carpet graphs. (C.-Ugurcan)

(iii) Regularity properties and level sets of continuous free fields on post-critically finite fractals, with applications to random dendrites. (C., in progress)

If time permits, I will discuss how these results can help answer physically inspired problems on fractals, such as: estimating the time that a random walk covers all vertices of a fractal graph; or determining whether a pinned random interface on a fractal substrate undergoes a wetting transition.

EVA CURRY, Acadia University
Poisson Boundary and Low-Pass Filters

Low-pass filters associated with scaling functions for multiresolution analysis wavelets satisfy a quadrature mirror condition: $|m(\xi/2)|^2 + |m(\xi/2+1/2)|^2 = 1$ a.e.. Given a starting point $\xi = \xi_0$, the values $|m(\xi/2)|^2$ and $|m(\xi/2+1/2)|^2$ can be considered as transition probabilities for a random walk/Markov process on the torus. A better way to think of this, however, is as a random walk down a binary tree, with root ξ_0, nodes $\xi_0/2$, $\xi_0/2+1/2$ on the first level, and so on.

The first half of the talk will introduce some notions of a boundary for this tree and random walk, including the hyperbolic boundary, Martin boundary, and Poisson boundary. Application of these tools in the study of wavelets and analysis on fractals is relatively new, so one aim of this talk it to improve awareness of the tools among the wavelets and fractals community. The second half of the talk will note some connections between characterization theorems for scaling functions (by Hernandez and Weiss) and for low-pass filters (by Gundy) and properties of the Poisson boundary. The results hold for multiresolution analyses in any dimension with any dilation matrix, but due to time constraints will only be stated in the one-dimensional, dilation by 2 case.

DYLAN DAY, Acadia

KATHRYN HARE, Dept. of Pure Mathematics, University of Waterloo
Multifractal analysis of self-similar measures with overlap

It is well known that the multifractal spectrum of a self-similar measure satisfying the open set condition is a closed interval. In contrast, Hu and Lau discovered the surprising fact that the 3-fold convolution power of the classical Cantor measure has an isolated point in its multifractal spectrum. More generally, this is true for any suitably large, convolution power of a continuous probability measure supported on $[0,1]$, which has the property that the local dimension at 0 is positive and the N-fold sum of the support of the measure is $[0,N]$ for some N. Self-similar measures generated by $m \geq d$ contractions, with fixed contraction factor $1/d$, $d \in N$, and probabilities $p_i > 0$ with p_0 minimal, also have an isolated point in their spectrum. If, however, some $p_i = 0$, the structure of the spectrum is more complicated and can even consist of two disjoint, non-trivial intervals.

CHUN-KIT LAI, McMaster University
Classification of self-affine tile digit sets as product-forms

Let A be an expanding matrix on $\mathbb{R}^s$ with integral entries. A fundamental question in the fractal tiling theory is to understand the structure of the digit set $\mathcal{D} \subset \mathbb{Z}^s$ so that the integral self-affine set $T(A,\mathcal{D})$ is a translational tile on $\mathbb{R}^s$. We first show that a tile digit set in $\mathbb{Z}^s$ must be an integer tile (i.e. $\mathcal{D} \oplus \mathcal{L} = \mathbb{Z}^s$ for some discrete set $\mathcal{L}$). We completely classify such tile digit sets $\mathcal{D} \subset \mathbb{Z}$ on $\mathbb{R}^1$ by expressing the mask polynomial $P_{\mathcal{D}}$ into product of cyclotomic polynomials and putting it in the trees of cyclotomic polynomials. This allows us to combine the technique of Coven and Meyerowitz on integer tiling on $\mathbb{R}^1$ to characterize explicitly all tile digit sets $\mathcal{D} \subset \mathbb{Z}$ with $A = p^\alpha q$ (p,q distinct primes) as $modulo\ product\text{-}form$ of some order, an advance of the previously known results for $A = p^\alpha$ and pq.

JOSHUA MACARTHUR, Dalhousie University
Wavelets with Crystal Symmetry Shifts

We introduce and explore the concept of wavelets when the lattice of shifts is replaced by a discrete group of measure preserving affine transformations of Euclidean space. This allows for all crystal symmetry groups to play the role of shifts, even the nonsymmorphic groups which cannot be handled with the existing theory of wavelets with composite dilations. Of particular importance to such Haar-type wavelets are their associated self-affine tiling set and the enumeration of the collection of shifts that yield convergence of the Barnsley-like iterated function system to said prototile(s) for all families of admissible dilations. Two dimensional examples will be presented. Joint work with Keith F Taylor.

FRANKLIN MENDIVIL, Acadia University
Geometry of a fractal curtain – stacking Cantor sets

In this talk we will explore some geometric properties of a beautiful set constructed by stacking central Cantor sets with continuously varying scaling factors. We find explicit formulas for the areas of all the gaps, find the Hausdorff and box-counting dimensions, and show that it consists of an uncountable number of smooth curves. Our derivation of the formulas for the gap areas is simple enough to be explained to first-year calculus students.

ROBERT G. NIEMEYER, University of New Mexico, Albuquerque
Dense orbits, periodic orbits and nontrivial paths of fractal billiard tables

In this talk, we will present examples of sequences of compatible periodic orbits of prefractal billiard tables. In particular, we will demonstrate the existence of such sequences for the Koch snowflake fractal billiard table, a self-similar Sierpinski carpet billiard table and the T-fractal billiard table. In each case, we will see that certain sequences of compatible periodic orbits exhibit interesting dynamical behavior and, in some cases, converge to periodic orbits. We will close by providing possible approaches to determining a wider class of recurrent orbits in each fractal billiard table and provide experimental evidence in support of the existence of 1) periodic orbits of fractal billiard tables and 2) dense orbits of fractal billiard tables. The material presented will be summarizing separate joint projects with M. L. Lapidus, R. L. Miller. and J. P. Chen.

BEN STEINHURST, Cornell University
Bond Percolation on the hexacarpet and related fractals

We discuss bond percolation on the a non-post critically finite analogue to the usual Sierpinski carpet and show that critical probability to percolate across the fractal is strictly less than one. Then using a modified dual graph argument we show that it is strictly greater than zero giving a non-trivial phase transition. The dual graph that arises is the hexacarpet which has recently been taken up as an interesting example. Our methods give a non-trivial phase transition on the hexacarpet as simple corollary to the main argument.

TARA TAYLOR, St. Francis Xavier University
Totally Disconnected Sierpinski Relatives

This talk will present work in progress regarding the totally disconnected Sierpinski Relatives. The Sierpinski Relatives are a class of fractals that include the well-known Sierpinski Gasket. They all have the same fractal dimension. Each is an attractor of an iterated function system (IFS) that involves three contractive similarities composed with symmetries of the square. In general, the relatives have different topologies. A sub-class of these fractals consists of the relatives that are totally disconnected. Current work focuses on the use of morphisms between fractals to distinguish between relatives. For example, some relatives have double points (points that correspond to two different addresses) while other ones do not.

JOZSEF VASS, University of Waterloo
Fractal Geometry via Containment, and the Exact Convex Hull of C-Type IFS Fractals

There are various directions of research currently under the label "Fractal Geometry", some well-established and others in development. Striving to adhere to the idea of Geometry in the classical sense, we present a philosophical approach which has been scatteredly emerging, meanwhile bridging the theoretical and the computer graphics literature, namely "Fractal Geometry via self-similar containment". We will discuss various computational methods and theoretical results that hinge on containing sets, such as bounding circles/spheres and the convex hull, essentially arguing that "Fractal Geometry" is in fact primarily about "containment", or the determination of containing sets. Thus we reason that finding the exact convex hull of an IFS fractal must be our focal quest, as the foundation for further geometrical investigations. In particular, we present a novel method for finding the exact convex hull of a C-type IFS fractal.