Pseudogroups and their Applications
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(Org: **Abraham Smith** (Fordham University) and/et **Francis Valiquette** (Dalhousie))

STEPHEN ANCO, Brock University
A group foliation method for finding exact solutions to nonlnear PDEs

In this talk I will outline a novel symmetry-group method which has been used successfully in recent work to find exact solutions to multi-dimensional wave equations and heat equations with power nonlinearities. The method is based on the geometrical idea of group foliation in which the solution jet space of a given nonlinear PDE is reduced to a quotient space of orbits under the action of a one-dimensional group of symmetries admitted by the PDE.Certain algebraic homogeneity features of the group-invariant equations describing the orbits are used to seek explicit solutions by a relatively simple separation ansatz.

ALEXANDER BIHLO, Centre de recherches mathématiques, Université de Montréal
Invariant and conservative parameterization schemes

Parameterization refers to the process of including unresolved subgrid-scale processes into a numerical model of the atmosphere-ocean system. In this talk we will introduce several methods for constructing parameterization schemes that preserve symmetries and conservation laws of systems of differential equations. We will illustrate these examples by constructing invariant and conservative closure schemes for the two-dimensional incompressible Euler equations and the shallow-water equations.

This talk is based on joint work with George Bluman and Roman O. Popovych.

DAVE MCNUTT, Dalhousie University
Vacuum Plane Waves: Equivalence and Physical Interpretation

In this talk I will give a brief summary of the Karlhede algorithm, and implement it for the gravitational plane wave spacetimes, as an illustrative example. This provides an invariant classification of these spacetimes in terms of a set of invariants - the so called Cartan invariants. By imposing conditions on the set of Cartan invariants one may reproduce known subclasses of these metrics. As a final point, I will show how the invariant structure relates to the physical interpretation of these spacetimes, by employing the geodesic deviation equations relative to timelike observers.

ROBERT MILSON, Dalhousie
A new characterization of the Painleve I equation

The invariant classification of second-order ordinary differential equation requires the calculation of differential invariants of the infinite-dimensional pseudo-group of point transformations. A basic question then becomes: what jet-order is required for invariant classification? For a given 2nd order equation $u_{xx} = q(x, u, u_x)$, we are able to show that the "worst-case" scenario involves 10th order jets of q and that that the Painlevé-I equation is precisely the simplest instance of such a maximal order equation. Our solution is based on the theory of equivariant moving frames and involves Cartan's notion of duality for 2nd order ODEs.

ALEXANDER ODESSKII, Brock University
Integrable matrix equations and algebraic structures related to them.

We discuss a special class of linear and quadratic Poisson brackets related to ODE systems with matrix variables. We investigate general properties of such brackets, present examples of compatible pairs of quadratic and linear brackets and find

the corresponding hierarchy of integrable models. An interesting class of compatible linear brackets is related to the affine Dynkin diagrams of A, D, E-type. Quadratic brackets are related with the so-called anti-Frobenius algebras.

ROMAN SMIRNOV, Dalhousie University
Algebraic aspects of orthogonal coordinate webs

We will discuss some algebraic aspects of the equivalence problem for the orthogonal coordinate webs defined by characteristic Killing tensors in spaces of constant curvature. This is joint work with Caroline Cochran.

ABRAHAM SMITH, Fordham University
Tableaux of PDE systems and associated Lie Pseudogroups

Lie Pseudogroups are defined as transformation groups whose elements are given as solutions of specific systems of involutive PDEs. By carefully analyzing the possible structures of the tableaux, symbols, and characteristics of those PDE systems, we expect to uncover a corresponding classification of Lie Pseudogroups.

ROBERT THOMPSON, University of Minnesota
Invariant submanifold flows and pseudogroup reconstruction

In recent work (2008), Peter Olver used invariant variational bicomplex formalism to develop general equations for the evolution of the differential invariant signature of a submanifold under an invariant submanifold flow. We recall these ideas and explain how to use a similar method to split an invariant submanifold flow into the evolution of an invariant signature together with a set of pseudogroup reconstruction parameters which map the evolved signature back to the evolved submanifold. Interesting applications included!

FRANCIS VALIQUETTE, Dalhousie University
Recursive Moving Frames

In the standard implementation of the equivariant moving frame method, differential invariants of a Lie pseudo-group action are constructed by first prolonging the action to the infinite submanifold jet space and then normalizing the pseudo-group parameters. Typically, this procedure yields unwieldy expressions that limit the method's practical scope. In the recursive approach the prolonged action is computed incrementally, and pseudo-group parameters are normalized as they appear to control, as much as possible, the expression swell. This is an ongoing project in collaboration with Peter J. Olver.