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Sign-changing solutions of the critical equation for the sublaplacian on the Heisenberg group

In this talk we will consider the subelliptic PDE

$$-\Delta_H u = |u|^{\frac{4}{Q-2}} u$$

on the Heisenberg group $\mathbf{H}^n$, where Δ_H is the sublaplacian operator on $\mathbf{H}^n$ and $Q = 2n+2$ is its homogeneous dimension. The differential operator is linear, second order, degenerate elliptic, and it is hypoelliptic being the sum of squares of (smooth) vector fields satisfying the Hormander condition.

The critical growth $\frac{Q+2}{Q-2}$ in the nonlinearity of the equation is related to a loss of compactness of the continuous embeddings of suitable anisotropic Sobolev-type spaces in standard L^p spaces (on bounded domains of $\mathbf{H}^n$), which occurs at the critical exponent $p^* = \frac{2Q}{Q-2}$.

Such an equation is related for instance to problems of differential geometry on Cauchy–Riemann manifolds, such as the CR-Yamabe problem of prescribing the Tanaka-Webster scalar curvature on $\mathbf{H}^n$ under a conformal change of the contact structure that identifies its CR structure.

We will show existence of infinitely many geometrically distinct sign changing solutions of the equation on $\mathbf{H}^n$ using variational techniques, exploiting the abundant symmetries of the equation and some notions of differential geometry in order to circumvent the lack of compactness of the energy functional, that arises at the considered critical growth.

This is a joint work with P. Mastrolia (Università degli Studi di Milano)