A conjecture of Andruskiewitsch and Dascalescu in 2004 stated that a Hopf algebra with non-zero integrals has finite coradical filtration. This was first observed by Radford in the ’70’s for pointed co-Frobenius Hopf algebras. Recently, a short and elegant proof for (a stronger version) of this conjecture was given by Andruskiewitsch, Cuadra and Etingof, as well as an extension to Frobenius tensor categories of subexponential growth. We extend their ideas to give a proof for a version of this statement in the setting of co-Frobenius coalgebras; in particular, some new examples of tensor categories where the Andruskiewitsch-Cuadra-Etingof result works are obtained. We also examine counter-examples for other versions of this conjecture for general co-Frobenius coalgebras.