If $G$ is a group, and $S$ is a commutative $G$-algebra, the Brauer-Clifford group $BrClif(S, G)$ is a group of equivalence classes of $S$-Azumaya $G$-algebras with a fixed nontrivial action of $G$ on $S$. These were introduced by Alexandre Turull in the context of his study of questions concerning various extensions of the McKay conjecture in the representation theory of finite groups.

Work of Dipra Mitra and the speaker showed that this Brauer-Clifford group is an equivariant Brauer group for the symmetric monoidal category of $S^*G$-modules for the skew group ring, where $S$ is any commutative $G$-algebra. This perspective allows one to define a Brauer-Clifford group in the category of $S\#H$-modules for the smash product when $H$ is a cocommutative Hopf algebra, and a dual Brauer-Clifford group involving an $H$-coaction when $H$ is commutative. I will introduce these new Brauer-Clifford groups and describe their basic properties. This is joint work with Thomas Guedenon.