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Surreal Analysis: An Analogue of Real Analysis for Surreal Numbers

Surreal numbers, which were discovered as part of analyzing combinatorial games, possess a rich numerical structure of their own and share many arithmetic and algebraic properties with real numbers. In order to develop the theory of surreal numbers beyond simple arithmetic and algebra, mathematicians have initiated the formulation of surreal analysis, the study of surreal functions and calculus operations. In this paper, we extend their work with a rigorous treatment of transcendental functions, limits, derivatives, power series, and integrals. In particular, we propose surreal definitions of three new analytic functions using truncations of Maclaurin series. Using a new representation of surreals, we present formulae for limits of sequences and functions (hence derivatives). Although the class of surreals is not Cauchy complete, we can still characterize the kinds of surreal sequences that do converge, prove the Intermediate Value Theorem, and establish the validity of limit laws for surreals. Finally, we show that some elementary power series and infinite Riemann sums can be evaluated using extrapolation, and we prove the Fundamental Theorem of Calculus for surreals so that surreal functions can be integrated using antidifferentiation. Extending our study to defining other analytic functions, evaluating power series in generality, finding a consistent method of Riemann integration, proving Stokes' Theorem to further generalize surreal integration, and solving differential equations remains open.