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The classification of uniserial $\mathfrak{sl}(2) \ltimes V(m)$-modules and a new interpretation of the Racah-Wigner $6j$-symbol

All Lie algebras and representations are assumed to be finite dimensional over $\mathbb{C}$. Let $V(m)$ be the irreducible $\mathfrak{sl}(2)$-module with highest weight $m \geq 1$ and let $\mathfrak{g}_m = \mathfrak{sl}(2) \ltimes V(m)$. In this talk we present a joint work with F. Szechtman in which we classify of all uniserial $\mathfrak{g}_m$-modules. Recall that a $\mathfrak{g}$-module is uniserial when its submodules form a chain. Uniserial modules are usually viewed as building blocks to understand more general classes of indecomposable representations. A classification of the indecomposable $\mathfrak{g}_m$-modules is far from being achieved even for $m = 1$, see [DR],[Pi].

In our classification, the main family of uniserial $\mathfrak{g}_m$-modules is actually constructed for any $\mathfrak{g} = \mathfrak{s} \ltimes V(\mu)$, where $\mathfrak{s}$ is a semisimple Lie algebra and $V(\mu)$ is the irreducible $\mathfrak{s}$-module with highest weight $\mu \neq 0$. It turns out that the members of this family are, but for a few exceptions of lengths 2, 3 and 4, the only uniserial $\mathfrak{g}_m$-modules.

One major step towards this classification is the determination of all admissible sequences of length 3, these are sequences $V(a), V(b), V(c)$ for which there is a uniserial $\mathfrak{g}_m$-module with these composition factors. This step depends in an essential manner on the determination of certain non-trivial zeros of Racah-Wigner $6j$-symbol.

References

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