MICHAEL COONS, University of Waterloo and Fields Institute

The rational-transcendental dichotomy of Mahler functions

In the late 1920s and early 1930s, Mahler wrote a series of articles concerning the algebraic character of values of power series which satisfy a certain type of functional equation; these functional equations (and functions) are now called Mahler-type functional equations (and Mahler functions). He was able to show that if a Mahler function $f(z)$ is transcendental then the number $f(a)$ is transcendental for all but finitely many nonzero algebraic numbers a in the radius of convergence of $f(z)$. Of course this result relies on the transcendence of a series, which may itself be difficult to ascertain. Some decades after Mahler's original investigations, Nishioka showed that a Mahler function was either transcendental or rational. Thus to show transcendence it is enough to show irrationality. In this talk, we will give a new (and much simpler) proof of Nishioka's theorem and discuss some refinements and generalizations.