Computation of Analytical Operators in Applied and Industrial Mathematics
Calcul des opérateurs analytiques en mathématiques appliquées et industrielles

(Org: **Peter Gibson** (York) and/et **Michael Lamoureux** (Calgary))

TIMUR AKHUNOV, University of Calgary

On the Cauchy problem for the Airy type equations

Dispersive equations historically emerged as models in the mathematical study of water waves starting with Laplace and Airy in the early 19th century. One of such equations is the third order Airy equation $\partial_t u + \partial_{xxx} u = 0$ in one space dimension. A key feature of the Airy equation is dispersion, which describes the propagation of localized waves to infinity. On the other hand, this dispersive effect is local and weakens at spatial infinity, where other effects may come into play. In this talk I will discuss a sharp condition on the coefficients of the equation for the wellposedness (existence, uniqueness, etc) of the Cauchy problem associated to Airy type equations $\partial_t u + \sum_{j=0}^{3} a_j(x)\partial_x^j u = 0$ with $C \leq a_3(x) \leq \frac{1}{C}$. The study of such Airy type equations is useful for non-linear problems that arise in water waves.

PETER GIBSON, York University

Identification of minimum phase preserving operators on the half line

Minimum phase functions are fundamental in a range of applications, including control theory, communication theory and signal processing. A basic mathematical challenge that arises in the context of geophysical imaging is to understand the structure of linear operators preserving the class of minimum phase functions. The heart of the matter is an inverse problem: to reconstruct an unknown minimum phase preserving operator from its value on a limited set of test functions. This entails, as a preliminary step, ascertaining sets of test functions that determine the operator, as well as the derivation of a corresponding reconstruction scheme. In the present paper we exploit a recent breakthrough in the theory of stable polynomials to solve the stated inverse problem completely. We prove that a minimum phase preserving operator on the half line can be reconstructed from data consisting of its value on precisely two test functions. And we derive an explicit integral representation of the unknown operator in terms of this data. A remarkable corollary of the solution is that if a linear minimum phase preserving operator has rank at least two, then it is necessarily injective.

CHUN-HUA GUO, University of Regina

Numerical solution of nonlinear matrix equations arising from Green's function calculations in nano research

The Green's function approach for treating quantum transport in nano devices requires the solution of nonlinear matrix equations of the form $X + (C^* + i\eta D^*)X^{-1}(C + i\eta D) = R + i\eta P$, where R and P are Hermitian, $P + \lambda D^* + \lambda^{-1} D$ is positive definite for all λ on the unit circle, and $\eta \to 0^+$. For each fixed $\eta > 0$, we show that the required solution is the unique stabilizing solution X_η. Then $X_* = \lim_{\eta \to 0^+} X_\eta$ is a particular weakly stabilizing solution of the matrix equation $X + C^* X^{-1} C = R$. In nano applications, the matrices R and C are dependent on a parameter, which is the system energy $\mathcal{E}$. In practice one is mainly interested in those values of $\mathcal{E}$ for which the equation $X + C^* X^{-1} C = R$ has no stabilizing solutions or, equivalently, the quadratic matrix polynomial $P(\lambda) = \lambda^2 C^* - \lambda R + C$ has eigenvalues on the unit circle. We point out that a doubling algorithm can be used to compute X_η efficiently even for very small values of η, thus providing good approximations to X_*. We also explain how the solution X_* can be computed directly using subspace methods such as the QZ algorithm by determining which unimodular eigenvalues of $P(\lambda)$ should be included in the computation.
(Based on joint work with Yueh-Cheng Kuo and Wen-Wei Lin.)

MIKE HASLAM, York University

Fast and Accurate Calculation of Acoustic Potentials with Application to Scattering from Periodically Rough Surfaces

We introduce highly accurate and efficient algorithms for the calculation of single and double layer acoustic potentials corresponding to the problem of scattering from parametric (possibly non-smooth) one-dimensional periodic surfaces. The scattering problem is formulated in terms of a second-kind surface integral equation. Left untreated, corner singularities in the surface profile seriously affect the convergence rate of the solution. We show that using appropriate regularization of the integral operators leads to arbitrarily high order asymptotic convergence rates for the surface density, and hence the scattered field. Numerical examples are given in the case of an incident plane wave with various wave numbers. With Oscar Bruno, Caltech.

SHAFIQUL ISLAM, University of Prince Edward Island (UPEI)
Computation of the Frobenius - Perron operator of dynamical systems

Densities of invariant measures for dynamical systems describe the long-time (statistical) behavior of the systems. In this talk, we present the Frobenius-Perron operator of dynamical systems as one of the main tools for the existence of invariant densities for dynamical systems. For random dynamical systems, we present some computational techniques of the Frobenius-Perron operator.

FOTINI LABROPULU, Luther College/University of Regina
Non-Newtonian mixed convection flowon a vertical surface in the presence of a magnetic field

The combined forced and free convection flow (mixed convection flow) is encountered in many technological and industrial applications which include solar central receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchangers placed in a low-velocity-environment, etc. The two-dimensional stagnation-point flow in a forced convection refers to the flow in the vicinity of a stagnation line that result from a two-dimensional flow impinging on a surface at right angles and flowing there after symmetrically about the stagnation line. The mixed convection in stagnation flow is important when the buoyancy forces due to the temperature difference between the surface and the free stream become large. Consequently, both the flow and thermal fields are significantly affected by the buoyancy forces. An analysis of the steady magnetohydrodynamics (MHD) mixed convection flow of a viscoelastic fluid stagnating orthogonally on a heated or cooled vertical flat plate has been studied. Using similarity variables, the governing equations are transformed into a system of two coupled non-linear ordinary differential equations. The resulting equations are then solved numerically using the spectral method.

MICHAEL LAMOUREUX, University of Calgary
Generalized frames for computing differential operators

In geophysical applications implementing numerical wavefield propagation, it has proven useful to decompose a complex geological medium into small local regions of nearly constant velocity, and propagate pieces of the wavefield through each region separately. The total wavefield is then obtained by reassembling all the pieces.

This decomposition/reassembling procedure can be described mathematically as a windowing procedure which is a specific implementation of generalized frames. By applying frame theory, we show that a collection of local wavefield propagators, combined via a suitable partition of unity, remains a stable propagator. This is a highly desirable property in numerical simulations and key to accurate solutions. These results apply more generally to combinations of linear operators that are useful for many nonstationary filtering operations.

ARTUR P SOWA, University of Saskatchewan
Operators resulting from the Dirichlet series

I will discuss a representation of the Dirichlet series in the form of infinite matrices, and its applications to the numerical and theoretical analysis of signals and operators. In particular, this structure plays a role in the construction of bases for $L_2[0, 2\pi]$ given by a set of dilated functions $\{f(nt) : n = 1, 2, \ldots\}$. Systems of dilated functions were investigated by A. Beurling in the 1940s and, after a long intermission, again in the 1990s in a paper by H. Hedenmalm, P. Lindqvist, and K. Seip. The Dirichlet-series matrix approach, on the other hand, was introduced quite recently, first in the context of nonlinear eigenvalue problems. It sheds new light at the analytic properties of these special bases. It also reveals that the corresponding basis transforms have an uncommon property of being numerically implementable via a lifting schema with $O(N \log N)$ efficiency. It is worthwhile mentioning that the connection of some such bases with the nonlinear oscillators indicates their strong applicability to the analysis of a plethora of signals, including signals characteristic for nanoelectronics. In addition, I will discuss an application of the Dirichlet-series matrices to the representation, analysis, and numerical manipulation of a broader class of operators.

MOHAMMAD TAVALLA, York University
A study on weighted composition operators on unit disk

We consider the set $\mathcal{W}$ of all weighted composition operators $M_\psi C_\phi$ acting on the space $\mathcal{A}$ of analytic functions on unit disk $\mathbb{D}$, and find necessary and sufficient conditions for a pair $(f,\ g)$ of analytic functions which can separate operators in this set. We also investigate representation of these pairs in terms of univalent analytic (Schlicht) functions on unit disk in order to find a full characterization of these pairs. This is a joint work with Professor Peter Gibson from departement of Mathematics at York University.

VLADIMIR ZUBOV, University of Calgary
Mesh refining in data preprocessing for the seismic inversion problem

Seismic inversion is a complex wave equation coefficients problem. Modern geophysical equipment provides researchers with high resolution images and seismograms which makes the wave data inversion more and more challenging. For acoustic waveform inversion standard gradient methods much easier convergences to approximate solutions on the coarse grid than on fine one. It gives geometrical multigrid method a chance to speed up the convergence and to optimize expenses on processing seismograms building the initial approximation of the velocity field. Just one iteration of multigrid with standard gradient method can be enough to prepare adequate approximation of the solution.

One of the main ingredients of multigrid are scaling of misfit function from grid to grid and solving a coarse problem. Local smoothing during the scaling is especially efficient in case of limited computational resources. On the other hand, in this situation the convergence of gradient method even more depends on the smoothness of the exact solution of the synthetic data.